Mon. Week 14

- WeBWorK: all past assignments are open
- Study Guide (Exam 4)

TRIG SUM FORMULAS

1 (c) $\sin(A+B) = \sin A\cos B + \sin B\cos A$

1 (d) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

MA 115 Exam 4(A) Name:

1. State the Law of Sines and Cosines:

(a) $\frac{\sin\alpha}{A} = \frac{\sin(B)}{B} = \frac{\sin(\gamma)}{C}$ (b) $A^2 = B^2 + C^2 - 2BC\cdot\cos(\alpha)$

you try →

2. Find the Exact Value (By hand)(Not showing work is worth 0 pts)

a) $\sin(\frac{7\pi}{12}) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}$

$= \sin\frac{\pi}{4}\cdot\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

b) $\cos(\frac{7\pi}{12}) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

$\tan(\frac{5\pi}{12}) = \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \frac{\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = \frac{\sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6}}{\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}} = \frac{\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\frac{A}{\frac{B}{C}} = \frac{A}{C}$

$\frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}-\sqrt{2}}\cdot\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) = \frac{6+2\sqrt{12}+4}{6-2} = \frac{10+2\sqrt{12}}{4}$

$= \frac{5+\sqrt{12}}{2}$

3. Find All Solutions to the Following:

a) $2\cos(4x) - \sqrt{3} = 0$

b) $\Big(1 + \tan(x)\Big)\sin(x) = 0$

c) $\cos(x)\sin(x) + \cos(x) = 0$

4. Verify the following identities:

a. $\dfrac{1+\cos(x)}{\sin(x)} + \dfrac{\sin(x)}{1+\cos(x)} = 2\csc(x)$ $\quad 2\cdot\left(\frac{1}{\sin(x)}\right)$

1. pick complicated side, use algebra to rewrite. : (common denom)

square binomial

$\dfrac{1+\cos x}{\sin(x)} \cdot \left(\dfrac{1+\cos(x)}{1+\cos x}\right) + \dfrac{\sin(x)}{1+\cos(x)} \cdot \dfrac{\sin(x)}{\sin(x)} = \dfrac{1+2\cos(x)+\underbrace{\cos^2(x)+\sin^2(x)}_{=1}}{(\sin(x))(1+\cos(x))}$

what is legal? what would be helpful

$\dfrac{2+2\cos(x)}{\sin(x)\cdot(1+\cos(x))}$

$\dfrac{2(1+\cos(x))}{\sin(x)(1+\cos(x))} = \dfrac{2}{\sin(x)} = 2\csc(x)$

b. $\dfrac{\cos^2(x)-\cos(2x)}{\sin^2(x)} = 1$ use trig sum formula to get a $\cos^2 x$ upstairs & $\sin^2 x$

$\cos(2x) = \cos(x+x) = \cos x \cdot \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$\dfrac{\cos^2(x) - (\cos^2 x - \sin^2 x)}{\sin^2 x} = \dfrac{\sin^2 x}{\sin^2 x} = 1$

Tools

- pythag. trig id
- common denom
- trig sum form

4. Solve for any possible triangles. If not possible, indicate so:

a) $a = 5$, $b = 7$, $c = 9$

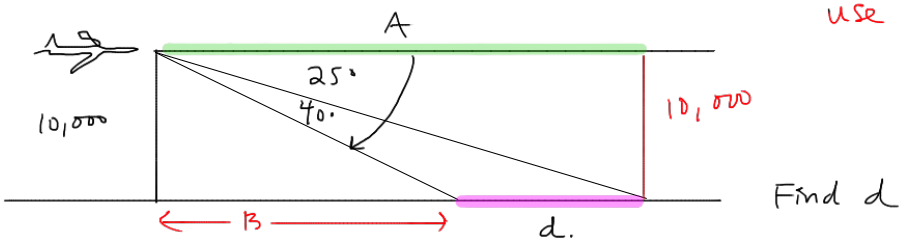
b) $A = 135, a = 5, b = 9$

c) $A = 15^\circ, a = 18, b = 32$

5. Word Problems

a) An incredibly intelligent and hungry pelican measures the angle of depression to two fish in the water in front of the themselves as 25 degrees and 40 degrees respectively. If the bird is flying at an altitude of 10,000 feet, help them find the distance between the two fish, so they can decide if they should try to eat both. Draw a picture.

use mixture:
1. Right Angle trig to get lengths A, B
2. law cosines to solve remainings.



b) Sir Arthur does not have enough time to cross the lake to fight the Dragon, so he must shoot it down with one well placed arrow.

To find the distance between himself and the Dragon, he sends his Scout who locates point S on land such that angle ASD is 55 degrees, the distance from Arthur to the Dragon is 279 feet, and the distance from the Scout to Dragon is 321 feet. Find the distance between the Dragon and Arthur. Draw a picture. (No need for a detailed dragon and knight...)

- similar to river problem in class

BONUS: Give me an example of a bonus question you think would be really fun to have on an exam.

Formula Sheet

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

$$\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

And as an application of these formulas...

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

And remember:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$