Thurs. Week 2

# Function Composition

$f(x) = \frac{1}{x-4}$

① $f(7) = \frac{1}{7-4} = \frac{1}{3}$ (evaluating functions)

$f(\text{any}) = \frac{1}{\text{any} - 4}$

② $f(f(x)) = \frac{1}{f(x) - 4} = \frac{1}{\left(\frac{1}{x-4}\right) - 4 \cdot \left(\frac{x-4}{x-4}\right)} \xrightarrow{\text{simplify}} \frac{1}{\frac{1 - 4x + 16}{x-4}}$

$= \frac{1}{\left[\frac{17-4x}{x-4}\right]} = 1 \cdot \frac{x-4}{17-4x} = \boxed{\frac{x-4}{17-4x}}$

## Basic Examples of Function Composition

1

$f(x) = x + 4$

$g(x) = \frac{1}{x^2}$

Notation: $f \circ g(x) = f(g(x))$

f composed w/ g

$f \circ g(x) = f(g(x)) = g(x) + 4 = \frac{1}{x^2} + 4$

$g \circ f(x) = g(f(x)) = \frac{1}{(f(x))^2} = \frac{1}{(x+4)^2}$

not the same (b/c $f \circ g(1) \neq g \circ f(1)$)

2

$f(x) = \frac{1}{x+3}$ $\quad g(x) = \frac{1-3x}{x}$

(we get $f \circ g(x) = x$ only because f and g are inverse functions)

$f \circ g(x) = f(g(x)) = \frac{1}{g(x)+3} = \frac{1}{\frac{1-3x}{x} + 3\left[\frac{x}{x}\right]} = \frac{1}{\frac{1-3x}{x} + \frac{3x}{x}} = \frac{1}{\frac{1-3x+3x}{x}} = \frac{1}{\left[\frac{1}{x}\right]} = x$

$g \circ f(x) = g(f(x)) = \frac{1 - 3 \cdot f(x)}{f(x)} = \frac{1 - 3\left(\frac{1}{x+3}\right)}{\left[\frac{1}{x+3}\right]} = \frac{\frac{x+3}{x+3} - \frac{3}{x+3}}{\left[\frac{1}{x+3}\right]} = \frac{\frac{x+3-3}{x+3}}{\left[\frac{1}{x+3}\right]} = \frac{x}{x+3} \cdot \frac{x+3}{1} = x$

Ex:

$$f(x) = \frac{1-5x}{2x-3}$$

find $f^{-1}(x)$ (inverse function)

$$x = \frac{1-5y}{2y-3}$$

cross-mult.

$$(2y-3)x = 1-5y$$

dist.

$$2yx - 3x = 1 - 5y$$

collect

$$2yx + 5y = 1 + 3x$$

$$y(2x+5) = 1+3x$$

$$y = \boxed{\frac{1+3x}{2x+5} = f^{-1}(x)}$$