Wednesday - Week 2

- WebWork: old set is re-opened — Sat 8 am.
- Today - Functions

warm-up: "solving quadratic-type equations" $(ax^2+bx+c=0)$ i.e., degree 2 poly

1. $x^6 - 7x^3 + 12 = 0$ [quadratic]

(a) recognize 2-1-3 pattern

(b) set: $u = x^3$, rewrite: (so $u^2 = x^6$ thus $u^2 - 7u + 12 = 0$

(c) $(u-3)(u-4) = 0$

(d) $u-3=0$ or $u-4=0$

(e) $u=3$ or $u=4$

(f) $x^3=3$ or $x^3=4$ ~~$x=\sqrt[3]{3}$~~ or $\sqrt[3]{4}$

OR $(x^3-3)(x^3-4)=0$

OR

AC

$x^6 - 3x^3 - 4x^3 + 12 = 0$

$x^3(x^3-3) - 4(x^3-3) = 0$

$(x^3-4)(x^3-3) = 0$

2. $\sqrt{x+1} = x$

$x+1 = x^2$ (square both)

$0 = x^2 - x - 1$

$ax^2+bx+c=0$

$\Downarrow$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2}$

$x = \frac{1 \pm \sqrt{1+4}}{2} = x = \frac{1 \pm \sqrt{5}}{2}$

golden ratio

Functions

(input – output machines ... a given input has exactly one output)

Ex. Your age is a function of time.

Ex Your mood is a funct of time.

Domain

set of allowable inputs.

Range:

all possible outputs

$f(x) = \frac{1}{x+1}$ has a restricted domain b/c $x+1 = 0$ is not allowed (b/c that gives $\frac{1}{0}$ which DNE)

throw out $x = -1$!

$(-\infty, -1) \cup (-1, \infty)$

$\left[\text{if } \frac{A}{0} = C \Rightarrow A = C \cdot 0 \Rightarrow A = 0 \text{ ⊗} \right]$

$g(x) = \sqrt{3x-6}$

note: "can't $\sqrt{\text{even}}$ take square root of a negative

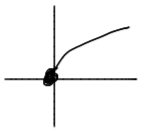
so: $3x - 6 \geq 0$

$3x \geq 6$

$x \geq 2$ set notation

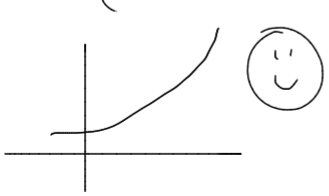
$[2, \infty)$ interval notation

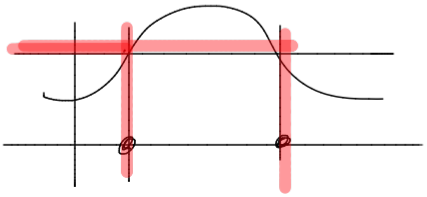
Range $[0, \infty)$



Inverses

Some functions can be reverse engineered (or made to go backwards)





Find the inverse function:

$f(x) = \frac{1}{x+3}$

① set $y = f(x)$ $\quad y = \frac{1}{x+3}$

② swap: $x \leftrightarrow y$ $\quad x = \frac{1}{y+3}$

③ solve for y

(i) cross mult. $\quad (y+3)x = 1$

(ii) distribute mult out $\quad yx + 3x = 1$

(iii) isolate y $\quad yx = 1 - 3x$

✓ + or −

✓ * or ÷ $\quad y = \frac{1-3x}{x}$

$f^{-1}(x) = \frac{1-3x}{x}$

$f(0) = \frac{1}{0+3} = \frac{1}{3}$

$f^{-1}\left(\frac{1}{3}\right) = \frac{1-3\left(\frac{1}{3}\right)}{\frac{1}{3}}$

$= \frac{1-1}{\frac{1}{3}} = 0 \div 3$

$= 0$