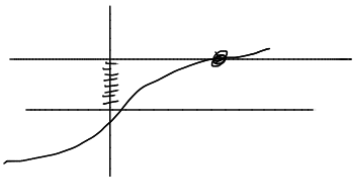
Monday - Week 3

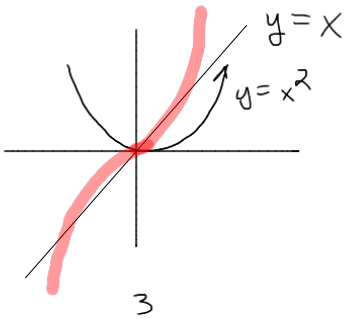
Functions	Geometry	Algebra
• Domain/Range • Inverses • Compositions	Lines: $y = mx + b$ $y - y_1 = m(x - x_1)$ slope parallel perpendicular	• Exponents • factoring • deg. 2 / grouping • AC-method • long div • quadratic formula • solving equations - quadratic type - radicals

Solving Radical Equations

① $\sqrt[3]{x-1} = 7$



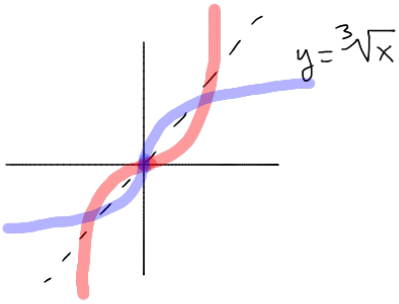
Imagine graphs of left side & right side



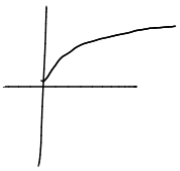
@ isolate x:

$$\left[\sqrt[3]{x-1}\right]^3 = 7^3 = 343$$

$$x - 1 = \boxed{x = 344}$$



② $\sqrt{x-1} + x = 3$



@ isolate the root

$$\sqrt{x-1} = 3 - x$$

$$(x-1) = (3-x)^2$$

(see if poly max. degree 2)
keep calm, expand quad. form

$$x - 1 = 9 - 6x + x^2$$

0 is special

$$0 = 10 - 7x + x^2$$

$$0 = x^2 - 7x + 10$$

$$0 = (x-2)(x-5)$$

$$x = 2 \quad \text{or} \quad x = 5$$

verify

$x = 2$: $\sqrt{2-1} + 2 = 3$ ✓

$x = 5$: $\sqrt{5-1} + 5 =$

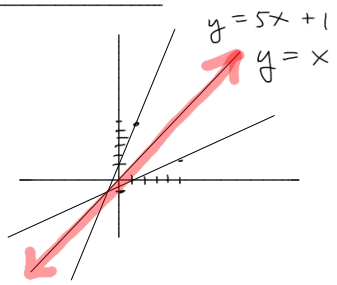
$\sqrt{4} = 2$

$2 + 5 = 7 \neq 3$ ✗

"$x = 3$ or $x = -4$"

Function Inverses

$f(x) = 5x + 1$



1. Set $y = 5x + 1$
2. swap $x = 5y + 1$
3. solve for y $x - 1 = 5y$

$\frac{1}{5}x - \frac{1}{5} = \frac{x-1}{5} = y =$

$$\boxed{f^{-1}(x) = \frac{1}{5}x - \frac{1}{5}}$$

Find the inverse.

Goal: isolate y.

Ex $f(x) = \frac{x-1}{x+1}$ | $y = \frac{x-1}{x+1}$ | $x = \frac{y-1}{y+1}$

a) cross-multiply
$x(y+1) = y-1$

b) distribute
$xy + x = y - 1$

c) collect like terms (use - subtraction)
$xy - y = -1 - x$

d) factor & divide
$y(x-1) = -1 - x$
$y = \frac{-1-x}{x-1}$

e) $f^{-1}(x) = \frac{-1-x}{x-1}$

$\frac{(-1)}{(-1)}\frac{(-1)(1+x)}{x-1}$

$= \frac{(1+x)}{(-1)(x-1)}$

$= \frac{1+x}{-x+1}$

$$\boxed{f^{-1}(x) = \frac{1+x}{1-x}}$$

Verify our answer:

we know

$f \circ f^{-1}(x) = x$

$f \circ f^{-1}(1) = 1$

$f \circ f^{-1}(-1) = -1$

check:

$f \circ f^{-1}(-1) = f\left(f^{-1}(-1)\right)$

$= f\left(\frac{0}{2}\right) = f(0) = \frac{0-1}{0+1} = -1$

verify

$f \circ f^{-1}(0) = f(-2)$

$f(-2) = 0$ ✓

Ex $f(x) = \sqrt[3]{2x+4}$

$y = \sqrt[3]{2x+4}$

$x = \sqrt[3]{2y+4}$ ... now solve

$x^3 = (x)^3 = \left(\sqrt[3]{2y+4}\right)^3 = 2y + 4$

$-4 \qquad\qquad -4$

$x^3 - 4 = 2y$

$\frac{1}{2}x - 2 \quad \frac{x^3-4}{2} = y$

$$\boxed{f^{-1}(x) = \frac{1}{2}x^3 - 2}$$