MA 115 Exam 1(A) Name:

1. Simplify the expression:

(a) $6y^0(3y^2)^{-1}$ $= 6 \cdot 1 \cdot \frac{1}{3y^2} = \boxed{\frac{2}{y^2}}$

(b) $\frac{4^2 b^3 c^{-3}}{(2a)^3 b^2 c^0}$ $= \frac{16b}{8a^3c^3} = \boxed{\frac{2b}{a^3c^3}}$

2. Completely factor the polynomial:

(a) $3x^2 + 7x - 6$ $= 3x^2 + 9x - 2x - 6$

$\longrightarrow$ AC: -18,

B: $7 = \underline{9} - \underline{2}$

$= 3x(x+3) - 2(x+3)$

$= \boxed{(3x-2)(x+3)}$

group

(b) $x^3 - 6x^2 - 4x + 24$

$x^2(x-6) - 4(x-6) = (x^2-4)(x-6)$

$= \boxed{(x-2)(x+2)(x-6)}$

3. Find all solutions to the equations:

(a) $x^6 - 7x^3 + 6 = 0$ $= x^6 - 6x^3 - 1\cdot x^3 + 6 = 0$

AC : 6

B : $-7 = -6 - 1$

$= x^3(x^3 - 6) - 1(x^3 - 6) = 0$

$= (x^3 - 6)(x^3 - 1) = 0$

So $x^3 - 6 = 0$ or $x^3 - 1 = 0$

$\boxed{x = \pm\sqrt[3]{6} \qquad x = \pm\sqrt[3]{1} = \pm 1}$

(b) $x - 5 = 4\sqrt{x}$

$(x-5)^2 = (4\sqrt{x})^2 = 4^2\sqrt{x}^2 = 16x$

$x^2 - 10x + 25 = 16x$

$x^2 - 26x + 25 = 0$

$x^2 - 25x - x + 25 = 0$

$x(x-25) - (x-25) = 0$

$(x-1)(x-25) = 0$

$\boxed{\cancel{x = 1}\ 25}$ $\boxed{x = 25}$

check! $x = 1 \Rightarrow$

$1 - 5 = 4\sqrt{1}$ NO!

$-4 = 4$ ⓧ

$x = 25$

$25 - 5 = 4\sqrt{25} = 4.5$ ☺

(c) $\sqrt[3]{2x+3} + 1 = 0$

This move saves time & likely errors

$\sqrt[3]{2x+3} = -1$

cube

$2x + 3 = (-1)^3 = -1$

$2x = -4$

$\boxed{x = -2}$ ✓

4. Find the Domain of the given functions:

(a) $f(x) = 13$

$\mathbb{R}$ — all real #'s

$(-\infty, \infty)$

$\{x \mid x \in \mathbb{R}\}$

(b) $f(x) = \dfrac{1}{x^2 - 3x}$

(can't divide by 0)

EXCLUDE

$x^2 - 3x = 0$

$x(x-3) = 0$

$\Rightarrow x = 0$

$x - 3 = 0, \ x = 3$

$\boxed{(-\infty,0)\cup(0,3)\cup(3,\infty)}$

or maybe

$\mathbb{R} - \{0, 3\}$

(c) $f(x) = \sqrt{4-x}$

No $\sqrt{}$ of a negative:

$4 - x \geq 0$ so $\boxed{4 \geq x}$

$[4, \infty)$

4 is included

∞ is not included

5. Write an equation for a line the satisfies the given characteristics:

slope:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

clear denom

$y_2 - y_1 = m(x_2 - x_1)$

point-slope

(a) passes through the points $\overset{x_1, y_1}{(5,2)}$ and $\underset{(x_2, y_2)}{(3,3)}$

$m = \dfrac{3-2}{3-5} = \dfrac{1}{-2} = \dfrac{2-3}{5-3} = \dfrac{-1}{2}$

$\boxed{y - 3 = \dfrac{1}{-2}(x-3)}$

or

$y = \dfrac{-1}{2}x + \dfrac{3}{2} + 3$

$\boxed{y = -\dfrac{1}{2}x + \dfrac{9}{2}}$

(b) passes through the points (-3,2) and (-3,7)

$m = \dfrac{7-2}{-3-(-3)} = \dfrac{5}{0} = \infty$ or DNE. } line is vertical

$\boxed{x = -3}$

(c) passes through (3,-2) perpendicular to $y = -\frac{1}{2}x - 6$

neg. recip

since $m = \dfrac{-1}{2}$, $m_\perp = 2$ ← our slope

$y - (-2) = 2(x - (3))$

or $\boxed{y + 2 = 2(x-3)}$

expand your answer

6. Find the following compositions of:

$f(x) = x^2 - 3x + 4$ and $g(x) = x - 3$

$f(x) = x^2 - 3x + 4$

(a) $f \circ g = f(g(x)) = (g(x))^2 - 3(g(x)) + 4$

$= (x-3)^2 - 3(x-3) + 4$

expand

$= x^2 - 6x + 9 - 3x + 9 + 4$

$= \boxed{x^2 - 9x + 22}$

(b) $g \circ g$

$g(g(x)) = g(x) - 3$

$= x - 3 - 3$

$= \boxed{x - 6}$

7. For each function find its inverse: set, swap, solve

(a) $f(x) = \sqrt[3]{x+5}$

set $y = \sqrt[3]{x+5}$

swap $x-y$: $x = \sqrt[3]{y+5}$

solve for y: "think outside-in"

$x^3 = \left(\sqrt[3]{y+5}\right)^3$ (1)

$x^3 = y + 5$ (2)

$x^3 - 5 = y$ (3)

$\boxed{f^{-1}(x) = x^3 - 5}$

(b) $f(x) = \dfrac{3x+2}{x-5}$

set $y = f(x) = \dfrac{3x+2}{x-5}$

swap $x = \dfrac{3y+2}{y-5}$

solve for y

(1) get everything on same level (clear denom)

$(y-5)x = 3y + 2$

(2) remove parenthesis (distribute)

$yx - 5x = 3y + 2$

(3) use $+,-$ to get terms w/ y on same side

$yx - 3y = 2 + 5x$

(4) $y(x-3) = 2 + 5x$

so $y = \boxed{\dfrac{2+5x}{x-3} = f^{-1}(x)}$

check:

$f^{-1} \circ f(0) =$

$= -\dfrac{2}{5}$

$\dfrac{2 + 5\left(-\frac{2}{5}\right)}{-\frac{2}{5} - 3} = \dfrac{2-2}{-\frac{17}{5}} = 0$