

FRIDAY - Week 6

writing an exponential function in terms of another base.

$A = P e^{rt}$  →  $A = P_0 a^t$

exponential  
function  
(base e)

Start  $A = P e^{rt}$   
 $= P (e^r)^t$  ↑  $a = e^r$

Notice  $P_0 = P$

$e^{rt} = e^{r \cdot t} = (e^r)^t$

$\begin{array}{l} \text{initial amt. } P = 1000 \\ \text{invest } \$1000 \text{ int' } \\ \text{account earnings } 5\% \text{ continuous interest} \\ \text{if } r = 5\% = .05 \\ \text{the amount this accumulates to is} \end{array}$

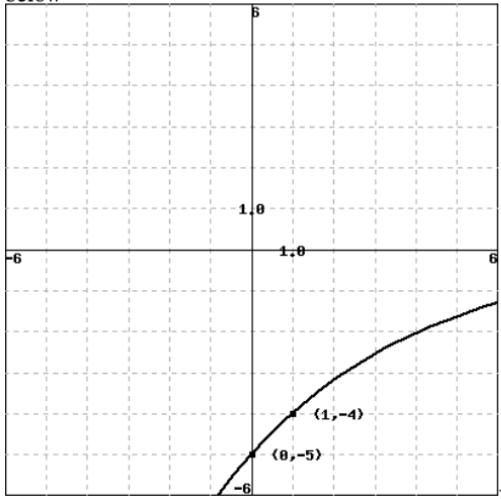
$\uparrow$   
initial value  
( b/c what this A is  
when t = 0 )

$$A = P e^{rt} \rightarrow A = 1000 \cdot e^{.05 \cdot t}$$

(when  $t = 0$ ,  $A = 1000 \cdot e^{.05 \cdot 0} = 1000 \cdot 1 = \$1000$ )

$t = 20$ ,  $A = (1000 \cdot e^{.05(20)}) = \$2718.$

Find the exponential function  $f(x) = a \cdot 2^{bx}$  whose graph is shown below



Know:  $(0, -5), (1, -4)$  live on the graph

$(x=0, f(x)=-5) \nparallel (x=1, f(x)=-4)$

make the equation true

plug in  $x=0$ , get  $-5$   
into  $f(x) = a \cdot 2^{bx}$

$$\begin{aligned} -5 &= a \cdot 2^{b \cdot 0} \\ &= a \cdot 2^0 \end{aligned}$$

$$-5 = a \cdot 1 = a$$

② Use what you learned in ①:

$$f(x) = (-5) \cdot 2^{bx} \neq -10^{bx}$$

use other data:  $(1, -4)$

$$-4 = (-5) \cdot 2^{b(1)}$$

$$\frac{-4}{-5} = 2^b$$

$$\boxed{\begin{array}{l} b/c \\ 2 \cdot 3^2 = 2 \cdot 9 = 18 \\ \times 6^2 \end{array}}$$

need to bring exponent down: hit w/ log

$$(i) \log_2\left(\frac{4}{5}\right) = \log_2(2^b) = b = -0.322$$

b/c these match

$$\boxed{f(x) = (-5) \cdot 2^{-0.322x}}$$

$$(ii) \ln\left(\frac{4}{5}\right) = \ln(2^b) = b \cdot \ln(2) \quad \text{solve for } b:$$

$$\frac{\ln(4/5)}{\ln(2)} = -0.322 = b$$

Note:

$$\boxed{\log_2\left(\frac{4}{5}\right)} = \text{something} = S$$

raise both sides  
as powers of 2

$$\log_2\left(\frac{4}{5}\right) = 2^S$$

$$\frac{4}{5} = 2^S$$

$$\ln\left(\frac{4}{5}\right) = \ln(2^S) = S \ln 2$$

$$S = \frac{\ln(4/5)}{\ln 2}$$

Find the exp. equation  $f(x) = P \cdot a^x$  passing through  $(1, 2)$  &  $(3, 4)$ .

use:  $(1, 2)$ :

$\begin{matrix} " \\ x \end{matrix}$   $\begin{matrix} " \\ f(x) \end{matrix}$

$$2 = P \cdot a^1 = P \cdot a, \text{ solve for } P:$$

$$\boxed{\frac{2}{a} = P}$$

use:  $(3, 4)$ :

$$4 = P a^3 = \left(\frac{2}{a}\right) \cdot a^3 = \frac{2}{a} \cdot \frac{a^3}{1} = \frac{2a^3}{a} = 2a^2$$

$$\begin{aligned} f(x) &= Pa^x = \frac{2}{a} (\sqrt{2})^x \\ &= \frac{2}{\sqrt{2}} (\sqrt{2})^x \\ &= \underline{\sqrt{2}} \cdot \underline{\sqrt{2}}^x \\ &= (\sqrt{2})^{x+1} \end{aligned}$$

$$\begin{array}{ccc} 2 = a^2 & \xrightarrow{\text{square root}} & \pm \sqrt{2} = a \\ a = \sqrt{2} & \xrightarrow{4} & \text{b/c exp. functions have positive bases! choose + root.} \end{array}$$

$$\frac{\underline{2}, \sqrt{2}}{\underline{\sqrt{2}}, \underline{\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$2 \cdot 3^2 \neq 6^2$$

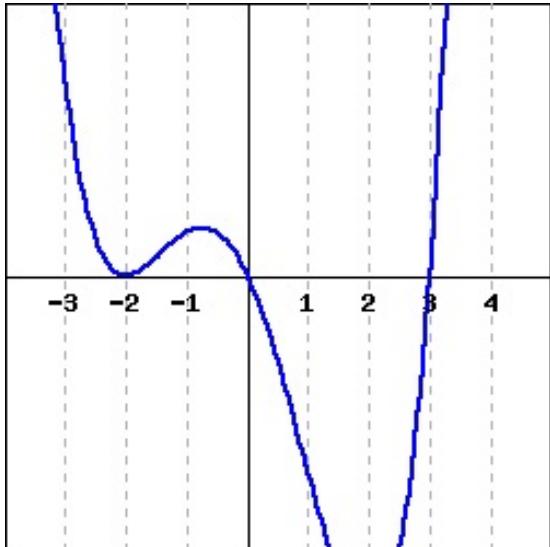
$$2 \cdot \underline{2}^4 = 2^5$$

Ex.

$$M(t) = (2 \cdot 5^t)^3 \xrightarrow{\text{write as}} m(t) = ab^t$$

$$\begin{aligned} &= 2^3 \cdot (5^t)^3 = 2^3 \cdot 5^{[t \cdot 3]} \\ &= 8 \cdot 5^{3t} \\ &= 8 \cdot (5^3)^t \end{aligned}$$

$$\begin{aligned} &= a = 8 \\ &b = 5^3 \end{aligned}$$



$$f(x) = k(x+2)^2(x-1)(x-3)$$

$$8 = k(-1+2)^2(-1)(-1-3)$$

$$= k(1)(-1)(-4)$$

$$= k(4)$$

$$2 = k$$

Find a degree 3 polynomial that has zeros  $-3, 4$  and  $5$  and in which the coefficient of  $x^2$  is  $-12$ .

The polynomial is  $x^3 - 7x^2 + 7x + 15$

$$k(x - (-3))(x - 4)(x - 5) = f(x)$$

$$k(x+3)(x-4)(x-5) = f(x)$$

$$k(x^2 - x - 12)(x-5)$$

$$k(x^3 - x^2 - 12x - 5x^2 + 5x + 60) = f(x)$$

$$k(x^3 - 6x^2 - 7x + 60) = f(x)$$

$$kx^3 - \cancel{6k}x^2 - 7kx + 60k = f(x)$$

$$-12 \Rightarrow k = 2$$