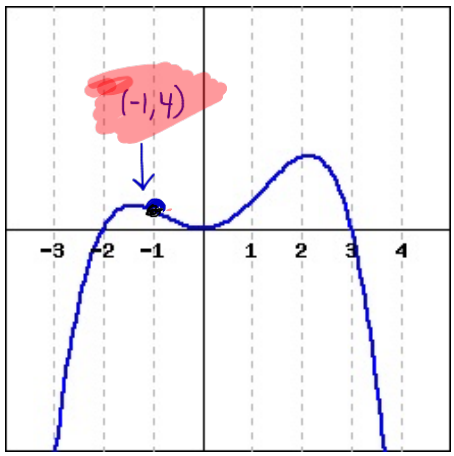
Monday – Week 6

WeBWork –



$f(x)$

polynomial: every one factors into linear terms

$f(x) = (x-a)(x-b)(x-c)\ldots(x-z)$

3 zeros → 4 degree poly

Two kinds of zeros: even degree ↔ graph kisses x-axis
odd degree ↔ graph pierces x-axis

pierce

degree = 1 b/c pierces

$$f(x) = k\cdot(x-(-2))(x-0)^2(x-3) = k(x+2)\cdot x^2\cdot(x-3)$$

term gives a zero

Knowing $(-1,4)$ is on graph means: [$x=-1$ forces $y=f(x)=4$]

$$4 = k(-1+2)(-1)^2(-1-3)$$

$$4 = k(-4)$$

$$-1 = \frac{4}{-4} = k$$

$$f(x) = (-1)(x^2)(x+2)(x-3)$$

#6/ Know $\{-1, 3, 5\}$ are the zeros & the x^2 - coef is -14.

<u>Start</u>: $f(x) = k\cdot(x-(-1))(x-3)(x-5)$

$= k\underbrace{(x+1)(x-3)}(x-5)$

$\boxed{f(x) = 2(x^3-7x^2+7x+15)}$

next expand $= k(x^2-2x-3)(x-5)$

$= k[(x^2-2x-3)x - 5(x^2-2x-3))]$

$= k[x^3-2x^2-3x-5x^2+10x+15)]$

$= k[x^3-7x^2+7x+15]$

$= kx^3 - (7k)x^2 + (7k)x + 15k$

Last use given:

x^2- coef $= -14$ so $-14 = -7k$ so $\boxed{k = 2}$

Finding Complete Factorizations (Finding all zeros _____)

Big Idea

Any polynomial:

$$f(x) = \underbrace{a}_{\text{leading coefficient}}\underbrace{x^n}_{\text{leading term}} + bx^{n-1} + cx^{n-2} + \dots + d$$

constant term

Any "rational zero" (a number that's a fraction of integers $1/2$, $-3/4$ [not $\sqrt{2}$]) of a poly will be

$\frac{p}{q}$ $\quad$ p is a factor of the constant term
$\quad$ q is a factor of the leading coef.

FOIL $\longrightarrow$

Ex $(2x-1)(x+1) = 2x^2 + 2x - x - 1 = 2x^2 + x - 1$

So $f(x) = 2x^2 + x - 1$

what are zeros: look for $\dfrac{\text{factors of } 1}{\text{factors of } 2}$

Possibilities: $\frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}$ i.e., $1, -1, 1/2, -1/2$

See: $f(1) = 2 + 2 - 1 = 3$

$f(-1) = 2 - 1 - 1 = 0$

$f(1/2) = 2(1/4) + 1/2 - 1 = 0$

$f(-1/2) = 2(1/4) - 1/2 - 1 \neq 0$

$2x^4 - 5x^3 + x^2 - 4. = f(x)$

Find all possible rational zeros & factor if possible.

$p = -4 \quad \} \; \pm 1, \pm 2, \pm 4$

$q = 2 \quad \} \; \pm 1, \pm 2$

$f(1) = 2 - 5 + 1 - 4 = -6$

$f(-1) = 2 + 5 + 1 - 4 = 4$

$f(\frac{2}{1}) = f(2) = 32 - 40 + 4 - 4 = -8$

$f(-2) = 32 - 5(-2)^3$

$= 32 + 40 + 4 - 4 \neq 0$

$f(4) =$

Ex

$f(x) = x^4 - 4$. Find all zeros.

all possible rational: $\dfrac{\text{factors of } 4}{\text{factors of } 1}$

$\pm 1, \pm 2, \pm 4$

$f(2) = 16 - 4 = 12$

$f(-2) = 12$

$f(1) = -3$

$f(-4) = 252$

No Rational Zeros

$x = \pm\sqrt{2}$

$x^2 = 2$

no real

Diff. of Squares: $f(x) = (x^2 - 2)(x^2 + 2)$

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