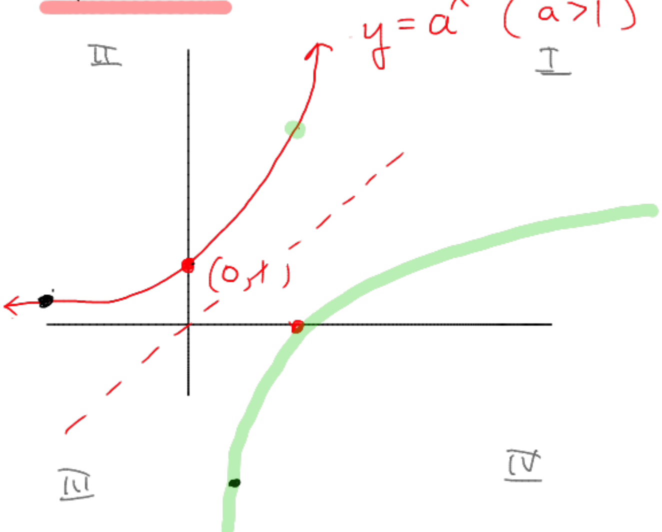
Thursday – Week 6

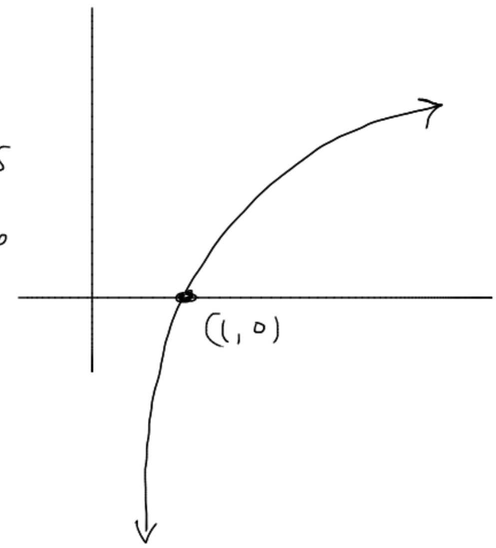
Logarithmic & Exponential Equations & Expressions – Ch. 6

Exp. Growth



reflections
⟷
about line
$y = x$

Logarithmic Growth



Def'n:

$$\log_a x = y \xrightarrow{\text{means}} a^y = x$$

Ex.

$$\log_2 8 = y \quad \text{means} \quad 2^y = 8 \quad \Rightarrow \quad y = 3$$

$$2 \cdot 2 \cdot \ldots \cdot 2 = 8$$

Logarithms are just exponents

PROPERTIES OF LOGS & EXPONENTIALS

EXP

$e^A \cdot e^B = e^{A+B}$

$(e^A)^C = e^{AC}$

$e^A + e^B$ doesn't simplify

Def'n of Log

$\log_a x = y$

means

$a^y = x$

$\ln x = \log_e x$

Log

- $\ln(A \cdot B) = \ln A + \ln B$
- $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$
- $\ln x^n = n \cdot \ln x$

How does one mathematician break-up w/ another?

$\ln(I'm) - \ln(You)$

$= \ln\left(\frac{I'm}{You}\right)$

- **Property 1:**

S for something unknown

start: $\ln_e(A \cdot B) = S$

apply def'n: $e^S = A \cdot B$.

next, give names to, then apply def'n, so combine:

$\ln A = \alpha \longrightarrow e^\alpha = A$

$\ln B = \beta \longrightarrow e^\beta = B$

$e^S = A \cdot B = e^\alpha \cdot e^\beta = e^{\alpha+\beta}$

$e^S = e^{\alpha+\beta}$

hit w/ ln (1-1 property)

$\Rightarrow \boxed{S = \alpha + \beta}$

go back to start

$\ln(A \cdot B) = S = \alpha + \beta = \ln A + \ln B$

Property 3

something

$\ln(x^n) = S$

def'n: $e^S = x^n$

apply n-th root to both

$e^{S(\frac{1}{n})} = (e^S)^{1/n} = (x^n)^{1/n} = x^{n/n} = x$

$\| \quad e^{S/n} = x$

log both

$\ln(e^{S/n}) = \ln x$

kill

$S/n = \ln x$

$S = n \cdot \ln x$

$\ln(x^n) = S = n \cdot \ln x$

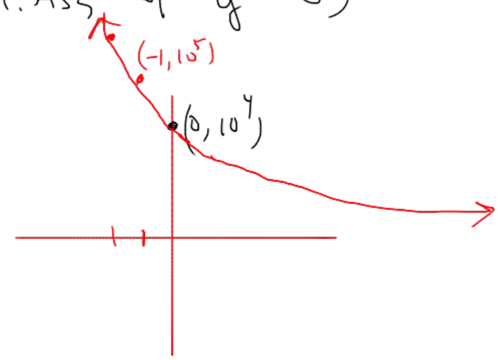
Ex

$f(x) = 10^{-(x-4)}$. Horiz. Asy? , Range , y-int?

$y = 0$

Range: set of all heights obtained by graph

$= \dfrac{1}{10^{(x-4)}}$ (can see H. Asy of $y = 0$)



$= \dfrac{1}{10^x \cdot 10^{-4}}$

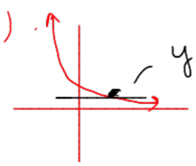
$y = a^{-x}$

exponential decay (negative exponent)

$= \dfrac{1 \cdot 10^4}{10^x} = \dfrac{10^4}{10^x}$

x	0	$x=-1$	$x=-2$	$x=1$
$f(x)$	$\frac{10^4}{10^0} = 10^4$	$\frac{10^4}{10^{-1}} = 10^5$		$\frac{10^4}{10^1} = 10^3$

Range: $(0, \infty)$.



$y = 1 = \dfrac{10^4}{10^x} \Rightarrow 10^x = 10^4$

$\log_{10}(10^x) = \log_{10}(10^4)$

$x = 4$

y-int: $x = 0$

plug in $x = 0$ $\rightsquigarrow$ $\dfrac{10^4}{10^0} = 10^4$