MA 115 Exam 2(A) Name: ____________________

1. Find all vertical and horizontal asymptotes:

   (a) $\dfrac{52}{x^2 - 5x + 6}$

   Horizontal Asymptotes:

   1500 degree is less than x^2 degree, so:

   $y = 0$

   Vertical Asymptotes:

   $x^2 - 5x + 6 = 0$

   $(x-3)(x-2) = 0$

   $x = 3, x = 2$

   (b) $\dfrac{x^2 - 2x - 3}{2x^2 - 5x - 3}$

   Horizontal Asymptotes:

   x^2 degree is equal to $2x^2$ degree, so:

   $y = \dfrac{1}{2}$

   Vertical Asymptotes:

   $2x^2 - 5x - 3 = 0$

   $2x^2 - 6x + x - 3 = 0$

   $2x(x-3) + 1(x-3) = 0$

   $(2x+1)(x-3) = 0$

   However $(x-3)$ is also a factor of the numerator so...

   $x = -\dfrac{1}{2}$ and NOT $x = 3$

   (c) $\dfrac{10 + x^3 - 7x^2}{x^3 + 11x^2 + 10x}$

   Horizontal Asymptotes:

   x^3 degree is equal to x^3 degree, so:

   $y = \dfrac{1}{1} = 1$

   $y = 1$

   Vertical Asymptotes:

   $x^3 + 11x^2 + 10x = 0$

   $x(x^2 + 11x + 10) = 0$

   $x(x+10)(x+1) = 0$

   However the Numerator cannot be factored so...

   $x = -10, -1, 0$

2. Completely factor the polynomial:

$$x^4 + 9x^3 + 22x^2 - 32$$

$$
\begin{array}{r|l}
 & x^3 + 5x^2 + 2x - 8 \\
\hline
x+4 & x^4 + 9x^3 + 22x^2 \qquad - 32 \\
 & \underline{-x^4 - 4x^3} \\
 & \quad 5x^3 + 22x^2 \\
 & \quad \underline{-5x^3 - 20x^2} \\
 & \qquad 2x^2 \\
 & \qquad \underline{-2x^2 - 8x} \\
 & \qquad\quad -8x - 32 \\
 & \qquad\quad \underline{8x + 32} \\
 & \qquad\qquad 0
\end{array}
$$

$$
\begin{array}{r|l}
 & x^2 + x - 2 \\
\hline
x+4 & x^3 + 5x^2 + 2x - 8 \\
 & \underline{-x^3 - 4x^2} \\
 & \quad x^2 + 2x \\
 & \quad \underline{-x^2 - 4x} \\
 & \qquad -2x - 8 \\
 & \qquad \underline{2x + 8} \\
 & \qquad\quad 0
\end{array}
$$

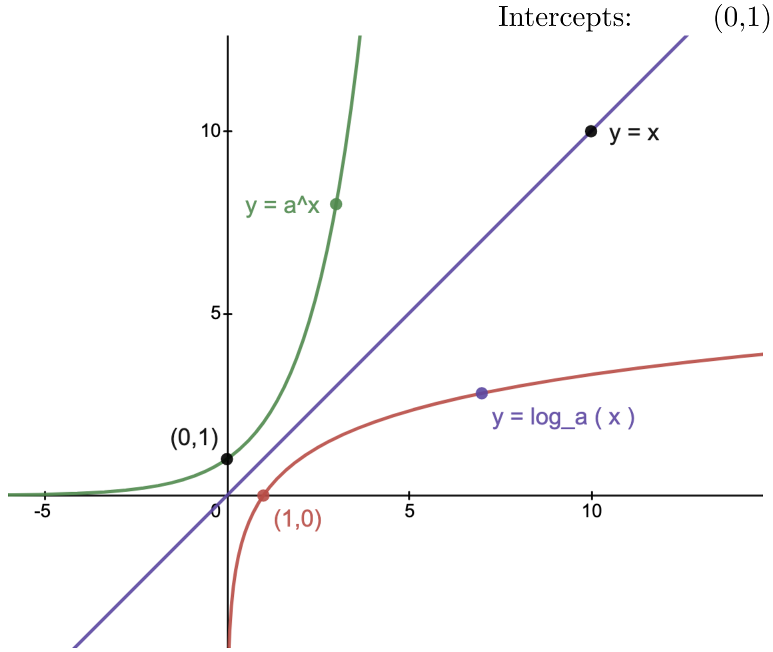
$$
\begin{array}{r|l}
 & x - 2 \\
\hline
x-1 & x^2 - 3x - 2 \\
 & \underline{-x^2 + x} \\
 & \quad -2x - 2 \\
 & \quad \underline{2x - 2} \\
 & \qquad -4
\end{array}
$$

$$= (x-1)(x+2)(x+4)(x+4)$$

3. Graph the functions, $y = a^x$ and $y = \log_a x$ on the same graph. Label any intercepts and give the domain and range of both. (If intercepts don't exist, write DNE)

Details about each graph

	$y = a^x$	$y = \log_a x$
Domain:	All Reals	$(0, \infty)$
Range:	$(0, \infty)$	All Reals
Intercepts:	(0,1)	(1,0)



4. Fill in the blank:

(a) $\ln(AB) = \ln(A) + \ln(B)$
(b) $\ln(\frac{A}{B}) = \ln(A) - \ln(B)$
(c) $\ln(e^x) = x$
(d) $e^{\ln(x)} = x$

5. Find the solution. (You should solve for the variable first, then grab a calculator)

(a) $e^{5x} = 15$
$\ln(e^{5x}) = \ln(15)$
$5x\ln(e) = \ln(15)$
$5x = \ln(15)$
$x = \frac{\ln(15)}{5}$
$x \approx .542$

(b) $5^{3x} = 9$
$\ln(5^{3x}) = \ln(9)$
$3x\ln(5) = \ln(9)$
$x = \frac{\ln(9)}{3\ln(5)}$
$x \approx .455$

(c) $3e^{4-x} = 8$
$e^{4-x} = \frac{8}{3}$
$\ln(e^{4-x}) = \ln(\frac{8}{3})$
$(4-x)\ln(e) = \ln(\frac{8}{3})$
$4 - x = \ln(\frac{8}{3})$
$x = 4 - \ln(\frac{8}{3})$
$x \approx 3.02$

(d) $4(6 + e^{2x}) = 27$
$(6 + e^{2x}) = \frac{27}{4}$
$e^{2x} = (\frac{27}{4} - 6)$
$\ln(e^{2x}) = \ln((\frac{3}{4}))$
$2x = \ln(\frac{3}{4})$
$x = \frac{\ln(\frac{3}{4})}{2}$
$x \approx -.144$

(e) $2\log(x) = \log(3) + \log(\frac{11}{3}x + 4)$

$\log(x^2) = \log(3) + \log(\frac{11}{3}x + 4)$

$\log(x^2) = \log(3(\frac{11}{4}x + 4))$

$\log(x^2) = \log(11x + 12)$

$10^{\log(x^2)} = 10^{\log(11x+12)}$

$x^2 = 11x + 12$

$x^2 - 11x - 12 = 0$

$(x - 12)(x + 1) = 0$

$x = 12, x = -1$

Check Solutions:

Can't have negative numbers... so $x \neq -1$

$x = 12$

(f) $\log_5 x + \log_5(x + 5) = \log_5 36$

$\log_5(x(x + 5)) = \log_5 36$

$\log_5(x^2 + 5x) = \log_5 36$

$5^{\log_5(x^2+5x)} = 5^{\log_5 36}$

$x^2 + 5x = 36$

$x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$x = -9, x = 4$

Check Solutions:

Can't have negative numbers... so $x \neq -9$

$x = 4$

6. Modeling. Answer the following:

(a) The number N of bacteria in a culture follows the exponential growth model, $N = Ae^{kt}$, where t is the time in hours. If the initial population is 400 and 3 hours later N = 1200, when will N = 2000?

Figure out A: (So plug in initial value at t = 0.)

$400 = Ae^{0k}$

$A = 400$

Next find the value of K by plugging in t = 3.

$1200 = 400e^{3k}$

$3 = e^{3k}$

$\ln(3) = \ln(e^{3k})$

$3k\ln(e) = \ln(3)$

$k = \frac{ln(3)}{3} \approx .366$

Now that we have the constants, we can find out when N = 2000:

$2000 = 400e^{.366t}$

$5 = e^{.366t}$

$\ln(5) = \ln(e^{.366t})$

$\ln(5) = (.366t)\ln(e)$

$.366t = \ln(5)$

$t = \frac{\ln(5)}{.366}$

$t \approx 4.39$ hours

(b) The population(p) of a mythical Badgermole, t years after it is introduced into a new habitat is given by:

$$p = \frac{4000}{1 + 3e^{-t/4}}$$

1. Determine the population size that was introduced into the habitat.
2. After how many years will the population be 2400?

1. Plug in 0 for t

$$p = \frac{4000}{1 + 3e^{0/4}}$$
$$p = \frac{4000}{1 + 3e^{0}}$$
$$p = \frac{4000}{1 + 3}$$
$$p = \frac{4000}{4}$$
$$p = 1000$$

2. Plug in 2400 for p

$$2400 = \frac{4000}{1 + 3e^{-t/4}}$$
$$2400(1 + 3e^{-t/4}) = 4000$$
$$(1 + 3e^{-t/4}) = \frac{4000}{2400}$$
$$(1 + 3e^{-t/4}) = \frac{5}{3}$$
$$3e^{-t/4} = \frac{5}{3} - 1$$
$$3e^{-t/4} = \frac{2}{3}$$
$$e^{-t/4} = \frac{2}{15}$$
$$\ln(e^{-t/4}) = \ln(\tfrac{2}{15})$$
$$\tfrac{-t}{4}\ln(e) = \ln(\tfrac{2}{15})$$
$$\tfrac{-t}{4} = \ln(\tfrac{2}{15})$$
$$-t = 4\ln(\tfrac{2}{15})$$
$$t = -4\ln(\tfrac{2}{15})$$
$$t \approx 8.06 \text{ years}$$