Thursday – Week 7

WW. #6 (Assis. #4)

$f(x) = a \cdot 2^{bx}$, given $(0,-5)$, $(1,-4)$ lie on the graph

determine a, b :

① $(0,-5)$: set $x = 0$, $f(x) = -5$ | $\underset{-5}{\overset{\shortparallel}{f(0)}} = a \cdot 2^{\overbrace{b \cdot 0}^{=0}} = a \cdot 2^0 = a \cdot 1 = a$

② UPDATE: $f(x) = -5 \cdot 2^{bx} \neq -10^{bx}$

③ $(1,-4)$: set $x = 1$, $f(x) = -4$ $\quad \underset{-4}{\overset{\shortparallel}{f(1)}} = -5 \cdot 2^{b \cdot 1} = \dfrac{-\cancel{5} \cdot 2^b}{-\cancel{5}} = \dfrac{-4}{-5}$

$$\Rightarrow \quad 2^b = \frac{-4}{-5} = \frac{4}{5}$$

$$\log_2(2^b) = \log_2(4/5)$$

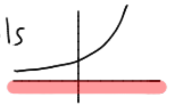
$$b = \log_2(4/5) = \frac{\ln(4/5)}{\ln(2)}$$

#12 $x^2 \cdot 5^x - 3x \cdot 5^x = 0$

get to $A \cdot B = 0$
then $A = 0$
or $B = 0$

$5^x(x^2 - 3x) = 0$

no sols

set $5^x = 0$ $\rightsquigarrow$ 

$\underbrace{x^2 - 3x}= 0$

$x(x-3) = 0$

$x = 0$

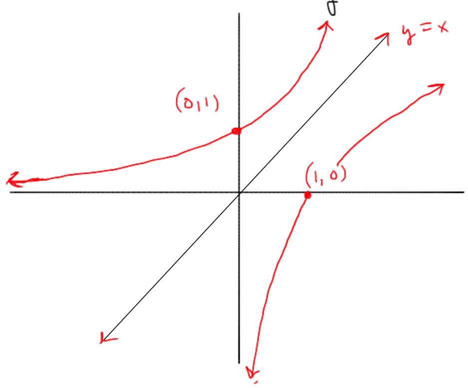
$x = 3$

#2 - set 5 1

$\log_5 \sqrt[4]{x^2+14} = A \cdot \log_5 f(x)$

$\log_5 (x^2+14)^{1/4} = \frac{1}{4} \log_5 (\overbrace{x^2+14})$

The log fcn & exp fcn relationship:



Key: a^x & $\log_a x$ are inverse functions

- their graphs are reflections of each across $y=x$
- domain(a^x) = range$(\log_a x)$ and range(a^x) = domain$(\log_a x)$

Range$(f(x))$ = set of #'s obtained as heights of the graph

Range$(a^x) = (0,\infty)$ = Domain$(\log_a x)$

Domain$(a^x) = (-\infty,\infty)$ = Range$(\log_a x)$

Ex. Set 6 # 3 $\quad f(x) = \log_a(x^2-16)$. What's domain?

(1) Domain of $\log_a x$ is $(0,\infty)$ so $x^2-16 \in (0,\infty)$

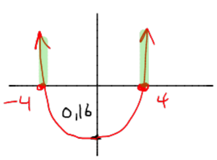
i.e.,

$$x^2 - 16 > 0$$

(2) Solve this inequality ↑.

Look at graph. Where is the graph above $y=0$

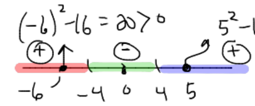
$(-\infty,-4) \cup (4,\infty)$



or

(i) set $x^2-16=0$ get $x^2=16$, $x=\pm 4$

(ii) $(-6)^2-16=20>0 \qquad 5^2-16>0$



Test values from each region

$(-\infty,-4) \cup (4,\infty)$

Set 5 #10

(1) $\ln A^c = c\cdot\ln A$

(2) $\ln(A\cdot B) = \ln A + \ln B$

(3) $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

(c) $\dfrac{\ln(a^2b^2)}{\ln((bc)^2)} =$

(d) $(\ln\ c^2)\left(\ln\frac{a}{b^3}\right)^4 =$

$$\frac{\ln(a^2)+\ln(b^2)}{2\cdot\ln(bc)} = \frac{2\ln(a)+2\ln(b)}{2\ln b+2\ln c} = \frac{2\cdot2+2\cdot3}{2\cdot3+2\cdot5} = \frac{10}{16} = \frac{5}{8}$$

(c)

$$\frac{\ln(a^{-4}b^3)}{\ln((bc)^1)} = \frac{\ln a^{-4}+\ln b^3}{\ln b+\ln c} = \frac{-4\ln a+3\ln b}{\ln b+\ln c} = \frac{-4\cdot2+3\cdot3}{3+5} = \frac{1}{8}$$

$$\frac{1+\cancel{4}}{\cancel{4}+6} = \frac{5}{10} = \frac{1}{2}$$

(c) $\dfrac{\ln(a^2b^2)}{\ln((bc)^2)} =$

(d) $(\ln\ c^2)\left(\ln\frac{a}{b^3}\right)^4 =$

$$2\cdot\ln c\cdot(\ln a-\ln b^3)^4$$

$$2\cdot\ln c(\ln a-3\ln b)^4 = 2\cdot(5)(2-3\cdot3)^4 = 10(7)^4 =$$

(b) The population(p) of a mythical Badgermole, t years after it is introduced into a new habitat is given by:

$$p = \frac{4000}{1 + 3e^{-t/4}}$$

logistic equation.

1. Determine the population size that was introduced into the habitat.

when $t = 0$,

2. After how many years will the population be 2400?

There's a steady state to the population.

what is p when $t = 0$? $\Rightarrow p = \underline{1000}$

what is p when $t = 1{,}000{,}000$? (approx) ≈ 4000

$$1 + 3e^{-t/4} = 1 + \frac{3}{e^{t/4}}$$