Wed. Week 7

WeBWork Questions

Problem 8. **(1 point)** `Library/LoyolaChicago/Precalc/Chap3Sec2/Q15.pg`

Find a formula for the exponential function that satisfies $h(2) = 20$ and $h(4) = 16.2$

① $\boxed{h(x) = Ca^x}$

② use given data to determine a, C

(i) $h(2) = 20$ means when $x = 2$ set everything equal to 20 $(h(x) = 20)$

$20 = h(2) = Ca^2$

(ii) solve for C: $\frac{20}{a^2} = C$

(iii) Sub this new "C" into $h(x)$:

$h(x) = Ca^x = \frac{20}{a^2} \cdot a^x$

(iv) use remaining data: $h(4) = \frac{20}{a^2} \cdot a^4 = 16.2$

(set $x = 4$
$Ca^x = 16.2$)

(v) solve for a: $\frac{20 \cdot a^4}{a^2} = 20 \cdot a^2 = 16.2$ so $a^2 = \frac{16.2}{20}$, $a = \sqrt{\frac{16.2}{20}} =$

(vi) now $C = \frac{20}{a^2} = \frac{20}{(16.2/20)} = \frac{\frac{20}{1}}{\frac{16.2}{20}} = \frac{20}{1} \cdot \frac{20}{16.2} = \frac{400}{16.2} =$

Any exponential function can be written as:

$\boxed{h(x) = C \cdot a^x}$

Ex: $f(x) = e^x$ is exponential
$c = 1$, $a = e \approx 2.718\ldots$

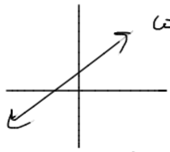
$f(x) = 2^x$, $c = 1$, $a = 2$

$f(x) = 1000(1 + .05)^{.7x}$
$c = 1000$
$a = (1 + .05)^{.7}$
$= 1000 \cdot \left[(1 + .05)^{.7}\right]^x$

{Base of exp. Function is always > 0} use + root

Distinguishing Between Linear & Exponential growth

Linear Functions: $f(x) = mx + b$



constant slope : constant rate of change : the Average Rate of Change, $\dfrac{f(B)-f(A)}{B-A}$ = constant.

$\boxed{\text{test for Linearity}}$

Ex $f(x) = 3x + 2$

x	4	8	12	20
$f(x)$	14	26	38	62

$$\frac{26-14}{8-4} = \frac{12}{4} = 3 \qquad \frac{38-26}{12-8} = \frac{12}{4} = 3$$

Exp. Functions $g(x) = Ca^x$

let $x = 2$, $\rightsquigarrow$ $g(2) = C\cdot a^2$

$x = 4$ $\rightsquigarrow$ $g(4) = C\cdot a^4$

$x = 6$ $\rightsquigarrow$ $g(6) = C\cdot a^6$

look @ ratio

ratio

$$\frac{C\cdot a^4}{C\cdot a^2} = a^2$$

$$\frac{Ca^6}{Ca^4} = a^2$$

ratio of outputs (for equal steps) are equal

$g(4) = 100$

$g(8) = 104.2$

If exponential:

$$\frac{Ca^8}{Ca^4} = \frac{104.2}{100} = 1.042$$

a^4

$$a = \sqrt[4]{1.042}$$

$$\frac{g(20)}{g(12)} = \frac{113{,}137}{108{,}576} = 1.042\ldots$$

$$\frac{Ca^{20}}{Ca^8} = a^4$$

Problem 13. **(1 point)** Library/Utah/Business_Algebra/set8_Exponential_and_Logarithmic_Functions/p21.pg

$\log_a x^c = c \cdot \log_a x$

$\boxed{\log_a(AB) = \log_a A + \log_a B}$

$\log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$

Let

$$L(x) = \log_a(x)$$

where we don't know the base a. However, we do know that

$$L(2) = 0.29721 \quad \text{and} \quad L(3) = 0.47107.$$

Use this information to compute

$L(4) =$ ____. $= L(2^2) = \log_a(2^2) = 2 \cdot \log_a(2) = 2L(2) = 2 \cdot (.29\ldots)$

$L(a^2) =$ ____. $2 \cdot L(a) = 2 \cdot \log_a(a)$

$L(a^3) =$ ____.

$L(6^5) =$ ____.

$\|$

$5 \cdot L(6) = 5 \cdot L(2 \cdot 3) = 5[L(2) + L(3)]$

$= 5[.29\ldots + .47\ldots]$

$\log_a x = c \iff a^c = x$

exponent required to make $a^c = x$

If $\ln(a) = 2$, $\ln(b) = 3$, and $\ln(c) = 5$, evaluate the following:

(a) $\ln\left(\frac{a^2}{b^{-2}c^{-3}}\right) =$ 25

$\ln(a^2) - \ln(b^{-2} \cdot c^{-3})$

$2 \cdot \ln(a) - [\ln b^{-2} + \ln c^{-3}]$

$2\ln(a) - [-2\ln b - 3\ln c]$

$2\ln a + 2\ln b + 3\ln c$

$2 \cdot 2 + 2 \cdot 3 + 3 \cdot 5 = 25$ ✓

(b) $\ln\sqrt{b^{-3}c^{-1}a^2} =$

(c) $\dfrac{\ln(a^2b^2)}{\ln((bc)^2)} =$

(d) $(\ln\ c^2)\left(\ln \frac{a}{b^3}\right)^4 =$

ln((stuff) ^(1/2)) = 1/2 * ln (b^(-3) * c^(-1) * a^2)

= 1/2 * (-3 ln(b) - ln(c) + 2 ln(a)   )