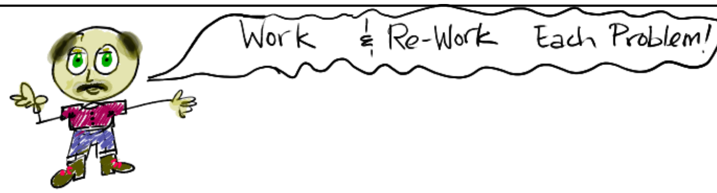


MA115 - Exam 2 Guide - October 14, 2018

1. **Evaluating Functions**
   Evalue the function below at $f(-5), f(0), f(1), f(2), f(5)$

   $f(-5) = -15$, $f(0) = 1$, $f(1) = 2$, $f(2) = 3$, $f(5) = 9$

$$f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x+1 & \text{if } 0 \le x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

$$\frac{f(b)-f(a)}{b-a}$$

2. Compute the average rates of change of the following functions on the given intervals

$f(x) = \dfrac{1}{x}$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ $\quad \dfrac{5-10}{\frac{1}{5}-\frac{1}{10}} = \dfrac{-5}{1/10} = -50$ $\quad$ $\underline{-50}$

$f(x) = \dfrac{1}{x}$ on the interval $[5, 10]$ $\quad \dfrac{\frac{1}{10}-\frac{1}{5}}{10-5} = \dfrac{-1/10}{5}$ $\quad$ $\underline{-1/50}$

$g(x) = x^2$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ $\quad \dfrac{\frac{1}{25}-\frac{1}{100}}{\frac{1}{5}-\frac{1}{10}} = \dfrac{\frac{3}{100}}{\frac{1}{10}}$ $\quad$ $\underline{3/10}$

$g(x) = x^2$ on the interval $[5, 10]$ $\quad \dfrac{100-25}{10-5} = \dfrac{75}{5} = 15$ $\quad$ $\underline{15}$

Discussion: How do these average rates of change relate to the end behavior of each the function $f(x) = \frac{1}{x}$ and $g(x) = x^2$?

as $x \to +\infty$, $f(x) \to 0$, the average rates of change decrease

as $x \to +\infty$, $g(x) \to 0$, the average rates of change increase.

3. Determine whether the following are odd, even or neither.

$f(x) = x^4 - 3x^2 + 5$

$f(-x) = (-x)^4 - 3(-x)^2 + 5$
$= x^4 - 3x^2 + 5$
$= f(x)$

even

$g(x) = \sqrt[3]{x}$

odd

$g(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -g(x)$

4. Find the domain of each of the functions

$f(x) = \dfrac{\sqrt{x-4}}{x^2+x-2}$

$x - 4 > 0$

discard: $x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = -2, x = 1$

$(4, \infty)$

$g(x) = \dfrac{1}{x - \dfrac{1}{1-x}}$

$1 - x \neq 0, \quad x \neq 1$

$x - \frac{1}{1-x} \neq 0$

$\Rightarrow x = \frac{1}{1-x}$

$x - x^2 = 1$

$0 = x^2 - x + 1$

$x = \frac{1 \pm \sqrt{1-4}}{2}$ no real sols

$(-\infty, 1) \cup (1, \infty)$

5. Find the inverse of the function

$y = f(x) = 6x^3 + 6$
swap
$x = 6y^3 + 6$

solve for y:
$x - 6 = 6y^3$
$\frac{x-6}{6} = y^3$ so $y = \sqrt[3]{\frac{x-6}{6}}$

$f^{-1}(x) = \sqrt[3]{\frac{x-6}{6}}$

Verify that what you found is indeed the inverse of f(x).

$f(f^{-1}(x)) = 6\left(\sqrt[3]{\frac{x-6}{6}}\right)^3 + 6 = 6\left(\frac{x-6}{6}\right) + 6 = x$ ✓

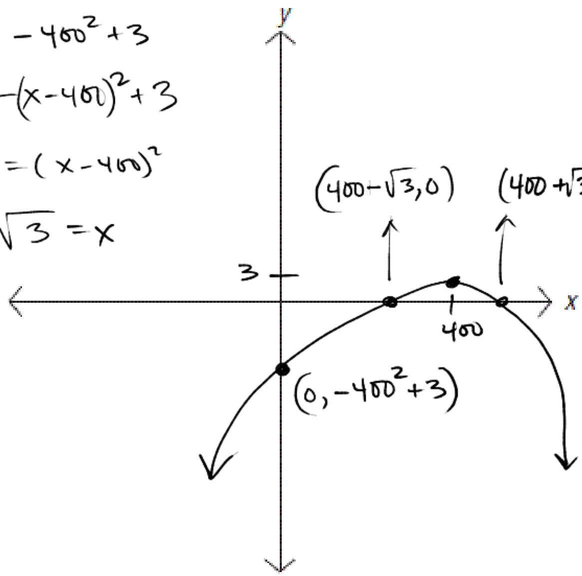
$f(0) = 200^2(-4)$

$f(0) = -400^2 + 3$

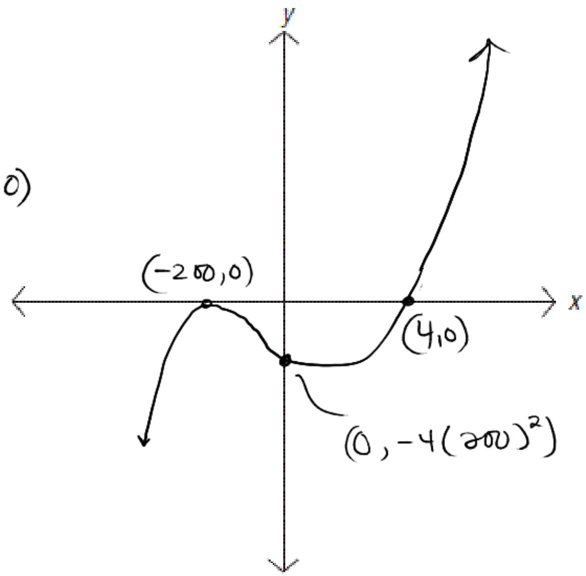
$0 = -(x-400)^2 + 3$

so $3 = (x-400)^2$

$400 \pm \sqrt{3} = x$



(a) $f(x) = -(x-400)^2 + 3$



(b) $f(x) = (x+200)^2(x-4)$

6. Sketch a graph of the functions above, clearly showing clearly all $x-$ and $y-$intercepts, and the vetex, if appropriate. Scale the axis as needed.

7. Find all the zeros of $f(x) = x^4 - 8x^3 + 11x^2 + 8x - 12$. You must show your work to get full credit.

$f(1) = 0$ } $x-1$ is a factor
$f(-1) = 0$ } $x+1$ is a factor } x^2-1 is a factor

$$
\begin{array}{r}
x^2 - 8x + 12 = (x-6)(x-2) \\
x^2-1 \overline{)\, x^4 - 8x^3 + 11x^2 + 8x - 12} \\
-(x^4 \qquad - x^2) \\
\hline
-8x^3 + 12x^2 \\
-(-8x^3 \qquad -8x) \\
\hline
12x^2 \qquad -12 \\
-(12x^2 \qquad -12) \\
\hline
0
\end{array}
$$

$x = 1, -1, 2, 6$

8. Consider the following rational functions:

$$r(x) = \frac{2x-1}{x^2-x-2}, \quad s(x) = \frac{x^3+27}{x^2+4}, \quad f(x) = \frac{x^3-9x}{x+2}, \quad g(x) = \frac{x^2+x-6}{x^2-25}$$

(a) Which of these rational functions has a horizontal asymptote?

$r(x)$, $g(x)$

@ $y=0$ @ $y=1$

(b) Which of these rational functions has a slant asymptote?

$s(x)$

@ $y=x$

(c) Which of these rational functions has no vertical asymptote? $s(x)$

9. Let $f(x) = \frac{x-1}{x+2}$ and $g(x) = \frac{1}{x}$.

$f(2) = \frac{2-1}{2+2} = \frac{1}{4}$

Compute $g(f(2)) =$ $g\left(\frac{1}{4}\right) = 4$ $\underline{\quad 4 \quad}$

Give a simplified expression for $f(g(x)) =$ $\underline{\quad \frac{1-x}{1+2x} \quad}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+2} = \frac{\frac{1-x}{x}}{\frac{1+2x}{x}} = \frac{1-x}{1+2x}$$

10. **Algebra of Logarithmic and Exponential Functions I**

Expand

$$\log\left(\frac{x(x^2+1)}{\sqrt{x^2-1}}\right) = \log(x) + \log(x^2+1) - \frac{1}{2}\left(\log(x-1) + \log(x+1)\right)$$

$$\ln\left(\sqrt{\frac{xe^x}{1+e^x}})\right) = \frac{1}{2}\left[\ln(x) + \ln(e^x) - \ln(1+e^x)\right]$$

$$= \frac{1}{2}\left[\ln(x) + x - \ln(1+e^x)\right]$$

Combine into a single logarithm

$$\ln(2(a+b)) + \ln(a-b) - 2\ln c = \ln\left(\frac{2(a^2-b^2)}{c^2}\right)$$

11. **Algebra of Logarithmic and Exponential Functions II**

Solve $\log(x) - \log(x-1) = \log(4x)$. $x = 5/4$

$\log\left(\frac{x}{x-1}\right) = \log 4x$

$\frac{x}{x-1} = 4x$

$\frac{1}{x-1} = 4$

$1 = 4x - 4$

$x = 5/4$

Solve $e^{2x} - e^x - 6 = 0$. $x = \ln 3$

$\underbrace{(e^x+2)}_{=0}\underbrace{(e^x-3)}_{=0} = 0$

$e^x = -2$ no sol's

$e^x = 3 \Rightarrow x = \ln 3$

Solve $10^{x+3} = 6^{2x}$.

$x + 3 = \log 6^{2x} = 2x \log 6$

$3 = 2x\log 6 - x = x(2\log 6 - 1)$

$x = \frac{3}{2\log 6 - 1}$

Solve $\log(x-5) + \log(x+3) = 1$. 6.05

$\log((x-5)(x+3)) = 1$

$(x-5)(x+3) = 10$

$x^2 - 2x - 15 = 10$

$x^2 - 2x - 25 = 0$

$x = \frac{2 \pm \sqrt{4 - 4 \cdot (-25)}}{2} = \frac{2 \pm \sqrt{104}}{2}$

$\approx \frac{2 \pm 10.1}{2} = 6.05$ or $\cancel{-4.05}$ not in domain

12. **Exponential Functions**
The population of the world was 5.7 billion in 1995 and the observed relative growth rate was 2% per year.

$A = Pe^{rt}$

(a) By what year will the population have doubled?

$1995 + 35$

2030

$2 = e^{.02t}$, $\ln 2 = .02t$, $t = \frac{\ln(2)}{.02} = 34.5 \approx 35$

(b) Using this model, what is the projected population in 2015?

$A = 5.7e^{.02(20)} = \underline{8.5 \text{ billion}}$

$A = Pe^{rt}$

13. The Ojibwe people of the Great Lakes region used birch bark scrolls to document history, songs, sacred rituals and maps. One such scroll was recently found to contain 94.7% of the Carbon-14 normally found in living birch bark. Approximately how old is this scroll?

$\frac{1}{2} = e^{r \cdot 5730}$

$\ln(1/2) = 5730 \cdot r$

$-0.00012 = \frac{\ln(1/2)}{5730} = r$

$.947 = e^{-0.00012t}$

$\frac{\ln(.947)}{-0.00012} = t = 453$ yrs