MA 115 Exam 2(A) Name:

1. Find all vertical and horizontal asymptotes:

(a) $\dfrac{52}{x^2 - 5x + 6}$

degree below > degree above $\Rightarrow$ Horiz. Asy @ 0: $y = 0$

(b) $\dfrac{x^2 - 2x - 3}{2x^2 - 5x - 3}$

degree below = degree above $\Rightarrow$ Horiz. Asy @ Ratio of leading terms $\dfrac{\cancel{x^2}}{2\cancel{x^2}}$ $\therefore$

$y = \frac{1}{2}$

(c) $\dfrac{10 + x^3 - 7x^2}{x^3 + 11x^2 + 10x}$

H.A. @ $y = 1$

2. Completely factor the polynomial:

$$x^4 + 9x^3 + 22x^2 - 32$$

1st find a zero: $x = a$ s.t. sub a in gives 0

Fact: look for $\pm \frac{p}{q}$ s.t. p = factor of 32 | 1, 2, 4, 8, 16, 32 ($\pm$)

q = factor of 1

$x=1 \Rightarrow 1^4 + 9(1) + 22 - 32 = 0$

2nd Long Division:

$$\begin{array}{r|l} & x^3 + 10x^2 + 32x + 32 \\ \boxed{x-1} & x^4 + 9x^3 + 22x^2 + 0x - 32 \\ & -(x^4 - x^3) \\ & 10x^3 \\ & -(10x^3 - 10x^2) \\ & 32x^2 \quad -32 \\ & -(32x^2 - 32x) \\ & 32x - 32 \\ & 32x - 32 \\ & 0 \end{array}$$

Now we try to factor, or find another zero: plugging into our "answer" or quotient from previous step

$P(1) = 1 + 10 + 32 + 32 \neq 0$

$P(-1) = -1 + 10 - 32 + 32 \neq 0$

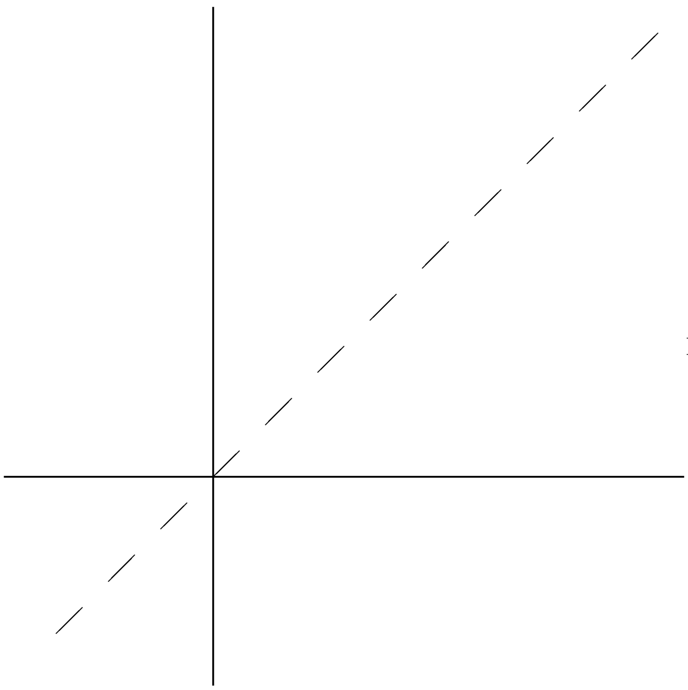
$P(2) = 8 + 40 + 64 + 32 = 0$

$P(-2) = -8 + 40 - 64 + 32 = 0$ ☺ $x - (-2) = x + 2$ is a factor of

$$\begin{array}{r|l} & x^2 + 8x + 16 \longrightarrow \boxed{(x+4)^2} \\ \boxed{x+2} & x^3 + 10x^2 + 32x + 32 \\ & -(x^3 + 2x^2) \\ & 8x^2 \\ & -(8x^2 + 16x) \\ & 16x + 32 \\ & 16x + 32 \\ & 0 \end{array}$$

$$\boxed{(x-1)(x+2)(x+4)^2}$$

3. Graph the functions, $y = a^x$ and $y = \log_a x$ on the same graph. Label any intercepts and give the domain and range of both. (If intercepts don't exist, write DNE)



Details about each graph

	$y = a^x$	$y = \log_a x$
Domain:		
Range:		
Intercepts:		

4. Fill in the blank:

(a) $\ln(AB) =$ ________________

(b) $\ln(\frac{A}{B}) =$ ________________

(c) $\ln(e^x) =$ ________________

(d) $e^{\ln(x)} =$ ________________

5. Find the solution. (You should solve for the variable first, then grab a calculator)

(a) $e^{5x} = 15$

(b) $5^{3x} = 9$

(c) $3e^{4-x} = 8$

(d) $\frac{4(6+e^{2x})}{4} = \frac{27}{4}$

Solve for x:
(work outside-in)

$6 + e^{2x} = \frac{27}{4}$

$e^{2x} = \frac{27}{4} - 6 = \frac{27}{4} - \frac{24}{4} = \frac{3}{4}$

$\ln(e^{2x}) = \ln\left(\frac{3}{4}\right)$

$2x = \ln\left(\frac{3}{4}\right)$ $\qquad x = \frac{\ln\left(\frac{3}{4}\right)}{2}$

(e) $2\log(x) = \log(3) + \log(\frac{11}{3}x + 4)$

(f) $\log_5 x + \log_5(x + 5) = \log_5 36$

6. Modeling. Answer the following:

(a) The number N of bacteria in a culture follows the exponential growth model, $N = Ae^{kt}$, where t is the time in hours. If the initial population is 400 and 3 hours later N = 1200, when will N = 2000?

$2000 = Ae^{kt}$
$= 400e^{[\frac{\ln 3}{3}] \cdot t}$ solve for t

$5 = \frac{2000}{400} = e^{(\frac{\ln 3}{3})t}$

$\ln 5 = \cancel{\ln (e}^{(\frac{\ln 3}{3}t)}$

$\ln 5 = \frac{\ln 3}{3} \cdot t$

$\boxed{\frac{3}{\ln 3} \cdot \ln 5 = t}$

$= 4.39$ hrs

- initial population means the number N when t = 0
- start w/ the question / what is being asked — ? → what is t when N = 2000 ? (3)

determine A
(1) N = 400 when t = 0 . ⇒ $400 = Ae^{k \cdot 0} = Ae^0 = A \cdot 1 = A$

(2) N = 1200 when t = 3 ⇒ $1200 = Ae^{k \cdot 3}$ (use (1)) ⇒ [÷ by 400] $3 = e^{3k}$ $\ln 3 = 3k$ $\boxed{\frac{\ln 3}{3} = k}$
determine k
$= 400 \cdot e^{3k}$

given

(b) The population(p) of a mythical Badgermole, t years after it is introduced into a new habitat is given by:

$$p = \frac{4000}{1 + 3e^{-t/4}}$$

1. Determine the population size that was introduced into the habitat. what is p when t = 0.
2. After how many years will the population be 2400? (what is t when p = 2400)

(1) $p = \frac{4000}{1 + 3e^{\cdot 0}} = 1000$ (now update formula)

(2) $\frac{2400}{1} = \frac{4000}{1 + 3e^{-t/4}}$ solve for t

cross-mult:

$24\cancel{00}(1 + 3e^{-t/4}) = 40\cancel{00}$

÷ by 24

$1 + 3e^{-t/4} = \frac{40}{24}$
$-1 \qquad -1$

still work outside-in

$3e^{-t/4} = \frac{40}{24} - \frac{24}{24} = \frac{16}{24} = \frac{2}{3}$

$e^{-t/4} = \frac{2}{9}$

$\ln(e^{-t/4}) = \ln(2/9)$

$-t/4 = \ln(2/9) = \ln(2) - \ln(9)$

mult. by -4

$t = -4(\ln(\frac{2}{9})) =$

↑ Notice a typo in posted sol's