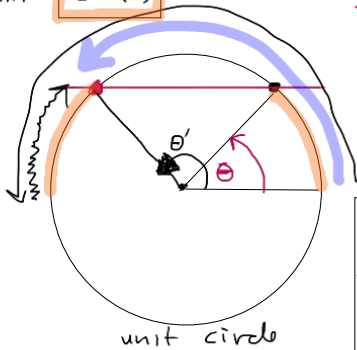


Friday - Week 9

Symmetry of trig. functions:

$f(x) = \sin(x)$

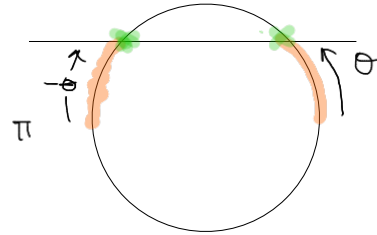


These y-coords are same ... the two angles have same sine.

To produce  $\theta'$  from  $\theta$   
 $\theta' = 180 - \theta$   
or  
 $\theta' = \pi - \theta$

ALWAYS TRUE

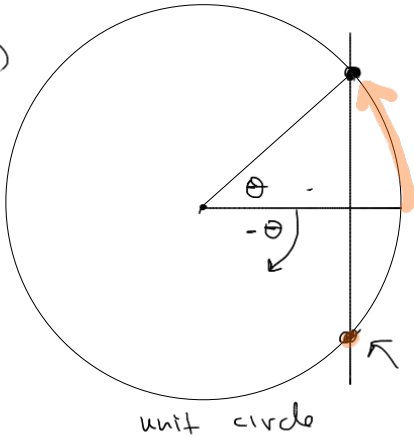
$$\sin(\theta) = \sin(\pi - \theta)$$



$f(\theta) = \cos(\theta)$

x-coord along unit circle

$[0, 2\pi)$

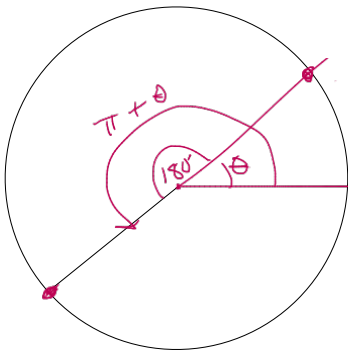


Same x-coord  $\Rightarrow$  same cosine

angle  $2\pi - \theta$

ALWAYS TRUE

$$\cos(\theta) = \cos(2\pi - \theta)$$



$\tan\theta = \text{slope of line det. by } \theta$

ALWAYS TRUE

$$\tan(\theta) = \tan(\pi + \theta)$$

Ex Find all sols in  $[0, 2\pi)$  of  $\sin(x) = .4$

$$\sin(x) = .4$$

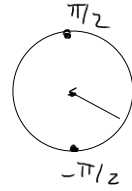
$$\sin^{-1}(\sin(x)) = \sin^{-1}(.4)$$

$$x = \sin^{-1}(.4)$$

$$x \approx 23.5^\circ$$

$$x' = 180 - 23.5 = 156.5^\circ$$

NOTE:  $\sin(x)$  is 1-1 on  $[-\frac{\pi}{2}, \frac{\pi}{2}]$   
 Domain  $\uparrow$   
 Range of  $\sin^{-1} = [-\frac{\pi}{2}, \frac{\pi}{2}]$



$\pi - \theta$

In radians:  
 $23.5 \times \frac{\pi}{180} = .415$   $\frac{1}{2}$   $\pi - .415 = 2.73$

Ex Find all sols to  $\cos(x) = .3$

$$x = \cos^{-1}(.3) = 1.27$$

$$x' = 2\pi - \cos^{-1}(.3) = 5.02$$

Two sols in  $[0, 2\pi)$

ALL SOLS integers

$$x = 1.27 + 2\pi k$$

$$x' = 5.02 + 2\pi k$$

$k \in \mathbb{Z}$

"lives in"

( $k$  is an integer)

Ex Find all sols to  $\tan(x) = 5$

$\pi + \theta$

All sol's in  $[0, 2\pi)$

①  $x = \tan^{-1}(5)$

$$x' = \tan^{-1}(5) + \pi$$

②

ALL sol's

$$x = \tan^{-1}(5) + \pi k$$

$$x' = \tan^{-1}(5) + \pi + \pi k$$

$$= \tan^{-1}(5) + (k+1)\pi$$

$$\textcircled{4} \cos(2x) = .4$$

$$\textcircled{A} \quad \cos(2x) = .4$$

$$\cos^{-1}(\cos(2x)) = \cos^{-1}(.4)$$

$$2x = \cos^{-1}(.4)$$

(before dividing by 2 ...

other  
sol'n

$$2x = 2\pi - \cos^{-1}(.4)$$

then

$$2x = 1.16 + 2\pi k \rightsquigarrow x = \frac{1}{2}(1.16) + \frac{1}{2}(2\pi k)$$

$$2x = 2\pi - 1.16 + 2\pi k$$

...  
Finally, divide by 2

$$= .58 + \pi k$$

and

$$x = \frac{1}{2}(2\pi - 1.16 + 2\pi k)$$

$$= \pi - .58 + \pi k$$

$$= (\pi - .58) + \pi k$$

$$\overset{\circlearrowleft}{=} -.58 + \pi(k+1)$$