1. True or False?

FALSE	$\ln(x^2+3x+2) = \ln x^2 + \ln 3x + \ln 2$
TRUE	$\ln(x^2+3x+2) = \ln(x+2) + \ln(x+1)$
TRUE	$2^x = e^{x\ln 2}$
TRUE	$\sqrt{x+3} = (x+3)^{1/2}$
FALSE	$\frac{1}{x^4} = x^{-1/4}$
FALSE	$\sqrt{x^2+1} = x+1$
TRUE	$\sqrt{x^2+4x+4} = x+2$
FALSE	$\frac{1}{x^2+4} = \frac{1}{x^2} + \frac{1}{4}$

2. "Simplify"

b.

$$\frac{1}{\left(\frac{4}{5}\right)^{-2}} = \frac{16}{25}$$

f.

$$\left(\frac{2xy^2}{5a^{-1}b^{-1}}\right)^{-1} = \frac{5}{2abxy^2}$$

3. Solve each equation.

(a)

$$e^x = e^{x^2-2} \quad \rightarrow \quad x = -1, 2$$

(b)

$$4(3^x) = 20 \quad \rightarrow \quad x = \frac{\ln 5}{\ln 3}$$

(d)

$$7 - 2e^x = 5 \quad \rightarrow \quad x = 0$$

(h)

$$\mathbf{\log_2(x-5) = 3} \quad \rightarrow \quad x = 13$$

(i)

$$\mathbf{\ln(x-5) = 3} \quad \rightarrow \quad x = e^3 + 5$$

4. Find x.

a.

$$\log_2 32 = x \quad \rightarrow \quad x = 5$$

b.

$$\log_2 \frac{1}{4} = x \quad \rightarrow \quad x = -2$$

c.

$$\log_2 \sqrt[3]{2} = x \quad \rightarrow \quad x = \frac{1}{3}$$

d.

$$\log_2 \sqrt[3]{4} = x \quad \rightarrow \quad x = \frac{2}{3}$$

e.

$$\log_2 x = -3 \quad \rightarrow \quad x = \frac{1}{8}$$

f.

$$\log_2 x = 4 \quad \rightarrow \quad x = 16$$

g.

$$\log_2 x = \frac{1}{2} \quad \rightarrow \quad x = \sqrt{2}$$

h.

$$\log_2 x = -\frac{1}{3} \quad \rightarrow \quad x = \frac{1}{\sqrt[3]{2}}$$

5. Find $f^{-1}(x)$ – or explain why it doesn't exist.

(a)

$$f(x) = \frac{\sqrt{2x-5}}{8} \quad \rightarrow \quad x = \frac{\sqrt{2y-5}}{8} \quad \rightarrow \quad 8x = \sqrt{2y-5}$$

$$\rightarrow \quad 64x^2 = 2y - 5 \quad \rightarrow \quad 64x^2 + 5 = 2y \quad \rightarrow \quad y = \frac{64x^2+5}{2}$$

$$\boxed{f^{-1}(x) = \frac{64x^2+5}{2}}$$

(b)

$$f(x) = \frac{2x-4}{5+3x} \quad \rightarrow \quad x = \frac{2y-4}{5+3y}$$

$$x(5+3y) = 2y - 4$$

$$5x + 3xy = 2y - 4$$

$$3xy - 2y = -4 - 5x$$

$$y(3x-2) = -4 - 5x$$

$$\boxed{f^{-1}(x) = y = \frac{-4-5x}{3x-2}}$$

a few more algebra reminders . . . .

1. Find the domain. Write it in interval notation.
(a) Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, +\infty)$

$$f(x) = \frac{2}{(x+2)(x-3)}$$

(b) Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, +\infty)$

$$g(x) = \frac{x-1}{(x+2)(x-3)}$$

(c) Domain: $(-\infty, +\infty)$

$$h(x) = x^2 + 3x + 2$$

(d) Domain: $(-\infty, +\infty)$

$$j(x) = 5$$

(e) Domain: $(-\infty, 2]$

$$k(x) = \sqrt{2-x}$$

(f) Domain: $(-\infty, 4) \cup (4, +\infty)$

$$l(x) = \frac{x}{\sqrt[3]{x-4}}$$

2. Let $f(x) = x^2 + 1$ and $g(x) = 2x + 3$. Find . . .
(a) $g \circ f = g(f(x)) = g(x^2+1) = 2(x^2+1) + 3 = 2x^2 + 5$

(b) $f \circ g = f(g(x)) = f(2x+3) = (2x+3)^2 + 1 = 4x^2 + 12x + 10$

(c) $g \circ g = g(g(x)) = g(2x+3) = 2(2x+3) + 3 = 4x + 9$

(d) $f \circ f = f(f(x)) = f(x^2+1) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$

3. For the given function, find (and simplify)
(i) $f(x+h)$
(ii) $f(x+h)-f(x)$
(iii) $\frac{f(x+h)-f(x)}{h}$

(a) $f(x) = 3x+7$

$$f(x+h) = 3(x+h)+7 = 3x+3h+7$$

$$f(x+h)-f(x) = (3x+3h+7)-(3x+7) = 3h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{3h}{h} = 3$$

(b) $f(x) = x^2$

$$f(x+h) = (x+h)^2 = x^2+2xh+h^2$$

$$f(x+h)-f(x) = (x^2+2xh+h^2)-(x^2) = 2xh+h^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$

(c) $f(x) = \frac{1}{x+3}$

$$f(x+h) = \frac{1}{(x+h)+3} = \frac{1}{x+h+3}$$

$$f(x+h)-f(x) = \frac{1}{x+h+3} - \frac{1}{x+3} = \frac{(x+3)}{(x+h+3)(x+3)} - \frac{(x+h+3)}{(x+h+3)(x+3)}$$

$$= \frac{(x+3)-(x+h+3)}{(x+h+3)(x+3)} = \frac{-h}{(x+h+3)(x+3)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{-h}{(x+h+3)(x+3)}}{h} = \frac{\frac{-h}{(x+h+3)(x+3)}}{\frac{h}{1}} = \frac{-h}{(x+h+3)(x+3)} \cdot \frac{1}{h}$$

$$= \frac{-1}{(x+h+3)(x+3)}$$

a few basic algebra reminders . . . .

1. Find an equation of the line that satisfies the given conditions:

(a) passes through (-1,-1) and (3, 7)

Answer: $y = 2x + 1$.

(b) passes through (7, 2) and (5, 2)

Answer: $y = 2$.

(c) passes through (-1, 3) and (-1, 5)

Answer: $x = -1$.

(d) passes through (-2, -6) and is parallel to $y = 2x + 3$.

Answer: $y = 2x - 2$.

(e) passes through (4, 2) and is perpendicular to $y = 2x + 3$.

Answer: $y = -\frac{1}{2}x + 4$.

2. Simplify the expression below (no negative exponents, no compound fractions).

(a)

$$\left(\frac{3}{y}\right)^3 \left(\frac{y^2}{4}\right)^{-2} = \frac{3^3}{y^3} \cdot \left(\frac{4}{y^2}\right)^2 = \frac{27}{y^2} \cdot \frac{16}{y^4} = \frac{432}{y^7}$$

(b)

$$5x^{-2}(-2y^0)^3 = 5\left(\frac{1}{x^2}\right)(-2)^3 = \frac{5}{x^2}(-8) = -\frac{40}{x^2}$$

(c)

$$\frac{\frac{2}{x+2}}{\frac{3}{x-2}} = \frac{2}{x+2} \cdot \frac{x-2}{3} = \frac{2(x-2)}{3(x+2)} = \frac{2x-4}{3x+6}$$

(d)

$$\frac{\frac{x+4}{3}}{\sqrt{x^2+16}} = \frac{\frac{x+4}{3}}{\frac{\sqrt{x^2+16}}{1}} = \frac{x+4}{3} \cdot \frac{1}{\sqrt{x^2+16}} = \frac{x+4}{3\sqrt{x^2+16}}$$

3. Combine into a single logarithmic term.

(a)

$$\ln(x+2) - \ln(x-1) = \ln\left(\frac{x+2}{x-1}\right)$$

(b)

$$\ln(x+2) - \ln(x-1) + \ln(x+1) = \ln\left(\frac{x+2}{x-1}\right) + \ln(x+1) = \ln\left(\frac{(x+2)(x+1)}{x-1}\right)$$

(c)

$$\frac{1}{3}\ln x - \frac{1}{2}\ln y - 2\ln z = \ln(x^{1/3}) - \ln(y^{1/2}) - \ln(z^2) = \ln\sqrt[3]{x} - \ln\sqrt{y} - \ln z^2 = \ln\left(\frac{\sqrt[3]{x}}{z^2\sqrt{y}}\right)$$

4. Use the logarithm rules to "reverse" the process in #3.

(a)

$$\ln\left(a^2bc^3\right) = \ln(a^2) + \ln b + \ln(c^3) = 2\ln a + \ln b + 3\ln c$$

(b)

$$\ln\left(a^2 - b^2\right) = \ln[(a+b)(a-b)] = \ln(a+b) + \ln(a-b)$$

(c)

$$\ln\left(\frac{a^2+b^2}{ab}\right) = \ln(a^2+b^2) - \ln a - \ln b$$