Integral applications #1

1. Find the average value of each of the functions below on the interval $\left[0, \frac{1}{2}\right]$. Use a graphing utility to sketch a graph of each of the function on this interval.

(a) $f(x) = 1/\sqrt{1-x}$

$$\frac{1}{\frac{1}{2} - 0} \int_0^{1/2} \frac{1}{\sqrt{1-x}}\, dx = 2\left[-2\sqrt{1-x}\right]_0^{1/2}$$

$$= -4\sqrt{\frac{1}{2}} - \left(-4\sqrt{1}\right) = 4 - 2\sqrt{2} \approx 1.17$$

See separate file for graph.

(b) $f(x) = 1/\sqrt{1-x^2}$

$$\frac{1}{\frac{1}{2} - 0} \int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\, dx = 2\left[\sin^{-1} x\right]_0^{1/2}$$

$$= 2\sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}(0) = 2\left(\frac{\pi}{6}\right) - 0 = \frac{\pi}{3} \approx 1.05$$

See separate file for graph.

2. Sketch the region enclosed by the given curves. Use an integral to find the area enclosed.

(a) $y = x$, $y = x^2$

$$\text{Area} = \int_0^1 x - x^2\, dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0) = \frac{1}{6}$$

See separate file for graph.

(b) $y = x^2 - 2x$, $y = x + 4$

$$\text{Area} = \int_{-1}^4 (x+4) - \left(x^2 - 2x\right)\, dx = \int_{-1}^4 -x^2 + 3x + 4\, dx$$

$$\left[-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x\right]_{-1}^4$$

$$\left(-\frac{1}{3}(4)^3 + \frac{3}{2}(4)^2 + 4(4)\right) - \left(-\frac{1}{3}(-1)^3 + \frac{3}{2}(-1)^2 + 4(-1)\right)$$

$$\left(\frac{56}{3}\right) - \left(-\frac{13}{6}\right) = \frac{125}{6} \approx 20.83$$

See separate file for graph.