Integral applications #1

1. Find the average value of each of the functions below on the interval $\left[0, \frac{1}{2}\right]$. Use a graphing utility to sketch a graph of each of the function on this interval.

(a) $f(x) = 1/\sqrt{1-x}$

(b) $f(x) = 1/\sqrt{1-x^2}$

2. Sketch the region enclosed by the given curves. Use an integral to find the area enclosed.

(a) $y = x$, $y = x^2$

(b) $y = x^2 - 2x$, $y = x + 4$

Applications

1. Optimization

(a) A rectangular storage container with a closed top and square base is to have a volume of 9 cubic meters. Material for the base costs $3 per square meter. Material for the sides costs $6 per square meter. Material for the top costs $1 per square meter.

Find the cost and dimensions of the cheapest such container.

(b) Find the point on the line $y = 2x + 5$ that is closest to the point (2,0). Draw a sketch.

2. Find the volume of the solid obtained by revolving the region between the x-axis and the curve $y = 1/x$, $1 \le x \le 4$, around the x-axis.

3. Find the work done when a particle is moved along the x-axis from the origin to $x = 5$ by a force given by $f(x) = 2e^{-2x} + 1$.

4. Given the following, find $f(x)$.
   (a) $f(x) = 6x - 2$
   (b) $f'(2) = 5$
   (c) $f(1) = 3$

5. Use a linear approximation, $L(x) = f(a) + f'(a)(x-a)$, to estimate $(1.05)^4$.

Applications - Sections 4.4, 4.1, 3.9

1. Evaluate the following limits.

(a)], $\displaystyle\lim_{x\to 0} \frac{\sin^{-1} x}{x}$

(b) $\displaystyle\lim_{x\to +\infty} \frac{(\ln x)^2}{x}$

(c) $\displaystyle\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$

(d) $\displaystyle\lim_{x\to +\infty} \left(1 + \frac{2}{x}\right)^x$

2. Find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 - 3x^2 - 12x + 10$ on the interval [0,5].

3. Two people start from the same point. One walks east at 3 mi!h and the other walks north at 2 mi!h. How fast is the distance between the people changing after 15 minutes?