

Differentiation - Quotient Rule

Find $f'(x)$.

1. $f(x) = \tan x = \frac{\sin x}{\cos x}$

$$f'(x) = \frac{\cos x \cdot \cos x - \sin x (-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

2. $f(x) = \frac{x+1}{x+2}$

$$f'(x) = \frac{(x+2) - (x+1)}{(x+2)^2} = \frac{1}{(x+2)^2}$$

3. $f(x) = \frac{x^2}{\sin x}$

$$f'(x) = \frac{\sin x \cdot 2x + x^2 \cdot \cos x}{\sin^2 x}$$

4. $f(x) = \frac{x^3}{\cos x}$

$$f'(x) = \frac{\cos x \cdot 3x^2 + x^3 \cdot \sin x}{\cos^2 x}$$

5. $f(x) = \frac{\ln x}{x^4}$

$$f'(x) = \frac{x^4 \cdot \frac{1}{x} - \ln x \cdot 4x^3}{x^8} = \frac{x^3(1-4\ln x)}{x^8} = \frac{1-4\ln x}{x^5}$$

$$6. f(x) = \sec x = \frac{1}{\cos x}$$

$$f'(x) = \frac{\cos x \cdot 0 + \sin x}{\cos^2 x} = \tan x \cdot \sec x$$

$$7. f(x) = \frac{e^x + 1}{e^x - 1}$$

$$f'(x) = \frac{(e^x - 1)e^x - (e^x + 1)e^x}{(e^x - 1)^2} = \frac{e^x(e^x - 1 - (e^x + 1))}{(e^x - 1)^2} = \frac{-2e^x}{(e^x - 1)^2}$$

$$8. f(x) = \frac{\ln x}{x}$$

$$f'(x) = \frac{x(\frac{1}{x}) - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$9. f(x) = \frac{e^x}{x^3}$$

$$f'(x) = \frac{x^3 \cdot e^x - e^x \cdot 3x^2}{x^6} = \frac{x^2 \cdot e^x (x - 3)}{x^6} = \frac{e^x (x - 3)}{x^4}$$

$$10. f(x) = \cot x = \frac{\cos x}{\sin x}$$

$$f'(x) = \frac{\sin x \cdot (-\sin x) - \cos x \cdot \cos x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$