Antiderivatives 1

For problems 1 - 7, find the antiderivative. Check your answer.

$$1. \int \frac{1}{2}x^2 - 2x + 6\,dx = \frac{1}{2}\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2} + 6x + C = \frac{1}{6}x^3 - x^2 + 6x + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{6}x^3 - x^2 + 6x\right] = \frac{1}{6}(3x^2) - 2x + 6(1) = \frac{1}{2}x^2 - 2x + 6$$

$$2. \int x(2-x)^2\,dx = \int x(4-4x+x^2)\,dx = \int x^3 - 4x^2 + 4x\,dx$$

$$= \frac{x^4}{4} - 4\cdot\frac{x^3}{3} + 4\cdot\frac{x^2}{2} + C = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2\right] = \frac{1}{4}(4x^3) - \frac{4}{3}(3x^2) + 2(2x) = x^3 - 4x^2 + 4x$$

$$3. \int \sqrt[4]{x^3} + \sqrt[3]{x^4}\,dx = \int x^{3/4} + x^{4/3}\,dx = \frac{x^{7/4}}{7/4} + \frac{x^{7/3}}{7/3} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$$

Check:

$$\frac{d}{dx}\left[\frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3}\right] = \frac{4}{7}\left(\frac{7}{4}x^{3/4}\right) + \frac{3}{7}\left(\frac{7}{3}x^{4/3}\right) = x^{3/4} + x^{4/3} = \sqrt[4]{x^3} + \sqrt[3]{x^4}$$

$$4. \int 3e^x + 7\sec^2 x\,dx = \int 3e^x\,dx + \int 7\sec^2 x\,dx = 3\int e^x\,dx + 7\int \sec^2 x\,dx$$

$$= 3e^x + 7\tan x + C$$

Check:

$$\frac{d}{dx}\left[3e^x + 7\tan x\right] = 3e^x + 7\sec^2 x$$

$$5. \int 2\sqrt{x} + 6\cos x\,dx = \int 2x^{1/2} + 6\cos x\,dx = 2\cdot\frac{x^{3/2}}{3/2} + 6(\sin x) + C = \frac{4}{3}x^{3/2} + 6\sin x + C$$

Check:

$$\frac{d}{dx}\left[\frac{4}{3}x^{3/2}+6\sin x\right]=\frac{4}{3}\left(\frac{3}{2}x^{1/2}\right)+6\cos x=2x^{1/2}+6\cos x=2\sqrt{x}+6\cos x$$

6. $\displaystyle\int 4+3(1+x^2)^{-1}\,dx=\int 4+3\left(\frac{1}{x^2+1}\right)\,dx=4x+3\tan^{-1}(x)+C$

Check:

$$\frac{d}{dx}\left[4x+3\tan^{-1}(x)\right]=4(1)+3\left(\frac{1}{x^2+1}\right)=4+3(1+x^2)^{-1}$$

7. $\displaystyle\int 6x+\sin x\,dx=6\cdot\frac{x^2}{2}+(-\cos x)+C=3x^2-\cos x+C$

Check:

$$\frac{d}{dx}\left[3x^2-\cos x\right]=3(2x)-(-\sin x)=6x+\sin x$$

For problems 8 - 10, find $f(x)$.

8. $f'(x)=8x^3+12x+3,\ f(1)=6$

$$f(x)=\int 8x^3+12x+3\,dx=8\cdot\frac{x^4}{4}+12\cdot\frac{x^2}{2}+3+C=2x^4+6x^2+3x+C$$

$$6=f(1)=2(1)^4+6(1)^2+3(1)+C\longrightarrow C=6-2-6-3=-5$$

Answer: $f(x)=2x^4+6x^2+3x-5$

9. $f'(x)=(x^2-1)/x,\ f(1)=\frac{1}{2}$.

$$f(x)=\int(x^2-1)/x\,dx=\int x-\frac{1}{x}\,dx=\frac{1}{2}x^2-\ln|x|+C$$

$$\frac{1}{2}=f(1)=\frac{1}{2}(1)^2-\ln|1|+C\longrightarrow C=0$$

Answer: $f(x)=\frac{1}{2}x^2-\ln|x|$

10. $f''(x) = 4 - 6x - 40x^3,\ f'(0) = 1,\ f(0) = 2$

$$f'(x) = \int 4 - 6x - 40x^3\,dx = -10x^4 - 3x^2 + 4x + C$$

$$1 = f'(0) = -10(0)^4 - 3(0)^2 + 4(0) + C \longrightarrow C = 1$$

$$f(x) = \int -10x^4 - 3x^2 + 4x + 1\,dx = -2x^5 - x^3 + 2x^2 + x + C$$

$$2 = f(0) = -2(0)^5 - (0)^3 + 2(0)^2 + (0) + C \longrightarrow C = 2$$

Answer: $f(x) = -2x^5 - x^3 + 2x^2 + x + 2$