

Antiderivatives 2

Find the antiderivative (#'s 1 - 5).

$$\begin{aligned} 1. \int 6\sqrt{x} - \sqrt[6]{x} dx &= \int 6x^{1/2} - x^{1/6} dx = 6 \cdot \frac{x^{3/2}}{3/2} - \frac{x^{7/6}}{7/6} + C \\ &= 4x^{3/2} - \frac{6}{7}x^{7/6} + C = 4x\sqrt{x} - \frac{6}{7}x\sqrt[6]{x} + C \end{aligned}$$

$$\begin{aligned} 2. \int \frac{5 - 4x^2 + 3x^4}{x^3} dx &= \int 5x^{-3} - 4\left(\frac{1}{x}\right) + 3x dx = 5 \cdot \frac{x^{-2}}{-2} - 4 \ln|x| + 3 \cdot \frac{x^2}{2} + C \\ &= -\frac{5}{2x^2} - 4 \ln|x| + \frac{3}{2}x^2 + C \end{aligned}$$

$$3. \int \frac{3}{x\sqrt{x^2-1}} + 7e^x dx = \int 3\left(\frac{1}{x\sqrt{x^2-1}}\right) + 7e^x dx = 3 \sec^{-1} x + 7e^x + C$$

$$4. \int 2 \sec x \tan x + 3 \sin x dx = 2 \sec x + 3(-\cos x) + C = 2 \sec x - 3 \cos x + C$$

$$5. \int \sqrt{5} \csc^2 x + \sqrt{2} dx = \sqrt{5}(-\cot x) + x\sqrt{2} + C = -\sqrt{5} \cot x + x\sqrt{2} + C$$

Find $f(x)$ (#'s 6 - 7).

6. $f'(x) = 6 \sec x \tan x + e^x$, $f(0) = 4$

$$f(x) = \int 6 \sec x \tan x + e^x dx = 6 \sec x + e^x + C$$

$$4 = f(0) = 6 \sec(0) + e^0 + C = 7 + C \longrightarrow C = -3$$

Answer: $f(x) = 6 \sec x + e^x - 3$

7. $f''(x) = 6x - 8$, $f'(0) = 3$, $f(1) = 5$

$$f'(x) = \int 6x - 8 \, dx = 3x^2 - 8x + C$$

$$3 = f'(0) = 3(0)^2 - 8(0) + C \longrightarrow C = 3$$

$$f(x) = \int 3x^2 - 8x + 3 \, dx = x^3 - 4x^2 + 3x + C$$

$$5 = f(1) = (1)^3 - 4(1)^2 + 3(1) + C \longrightarrow C = 5$$

Answer: $f(x) = x^3 - 4x^2 + 3x + 5$

Use the suggested substitution to find the antiderivative (#'s 8 - 10). CHECK YOUR ANSWER.

8. Use $u = 3x + 4 \rightarrow du = 3 \, dx \rightarrow \frac{1}{3} \, du = dx$.

$$\int 3e^{3x+4} \, dx = \int 3e^u \left(\frac{1}{3}\right) \, du = \int e^u \, du = e^u + C = e^{3x+4} + C$$

Check:

$$\frac{d}{dx} [e^{3x+4}] = \frac{e}{dx} [3x + 4] e^{3x+4} = 3e^{3x+4}$$

9. Use $u = x^2 + 1 \rightarrow du = 2x \, dx \rightarrow \frac{1}{2x} \, du = dx$.

$$\int 2xe^{x^2+1} \, dx = \int 2x e^u \left(\frac{1}{2x}\right) \, du = \int e^u \, du = e^u + C = e^{x^2+1} + C$$

Check:

$$\frac{d}{dx} [e^{x^2+1}] = \frac{d}{dx} [x^2 + 1] e^{x^2+1} = 2x e^{x^2+1}$$

10. Use $u = x^3 \rightarrow du = 3x^2 \, dx \rightarrow \frac{1}{3x^2} \, du = dx$.

$$\begin{aligned} \int 3x^2 \sec(x^3) \tan(x^3) \, dx &= \int 3x^2 \sec u \tan u \left(\frac{1}{3x^2}\right) \, du = \int \sec u \tan u \, du \\ &= \sec u + C = \sec(x^3) + C \end{aligned}$$

Check:

$$\frac{d}{dx} [\sec(x^3)] = \frac{e}{dx} [x^3] \sec(x^3) \tan(x^3) = 3x^2 \sec(x^3) \tan(x^3)$$