

Antiderivatives 4

Find the indicated antiderivative. Check your answers.

1. Use $u = x^3 \rightarrow du = 3x^2 dx \rightarrow \frac{1}{3x^2} du = dx$

$$\int 5x^2 \cos(x^3) dx = \int 5x^2 \cos(u) \cdot \frac{1}{3x^2} du = \int \frac{5}{3} \cos u du = \frac{5}{3} \sin u + C = \frac{5}{3} \sin(x^3) + C$$

Check:

$$\frac{d}{dx} \left[\frac{5}{3} \sin(x^3) \right] = \frac{5}{3} \cos(x^3) 3x^2 = 5x^2 \cos(x^3)$$

2. Use $u = x^2 + 1 \rightarrow du = 2x dx \rightarrow \frac{1}{2x} du = dx$

$$\begin{aligned} \int x \sin(x^2 + 1) dx &= \int x \sin(u) \left(\frac{1}{2x} \right) du = \int \frac{1}{2} \sin u du \\ &= -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(x^2 + 1) + C \end{aligned}$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{2} \cos(x^2 + 1) \right] = -\frac{1}{2} (-\sin(x^2 + 1)) (2x) = x \sin(x^2 + 1)$$

3. Use $u = x^4 - 1 \rightarrow du = 4x^3 dx \rightarrow \frac{1}{4x^3} du = dx$

$$\begin{aligned} \int 2x^3 \sec(x^4 - 1) \tan(x^4 - 1) dx &= \int 2x^3 \sec(u) \tan(u) \left(\frac{1}{4x^3} \right) du = \int \frac{1}{2} \sec u \tan u du \\ &= \frac{1}{2} \sec u + C = \frac{1}{2} \sec(x^4 - 1) + C \end{aligned}$$

Check:

$$\begin{aligned} \frac{d}{dx} \left[\frac{1}{2} \sec(x^4 - 1) \right] &= \frac{1}{2} \sec(x^4 - 1) \tan(x^4 - 1) (4x^3) \\ &= 2x^3 \sec(x^4 - 1) \tan(x^4 - 1) \end{aligned}$$

4. Use $u = x^3 \rightarrow du = 3x^2 dx \rightarrow \frac{1}{3x^2} du = dx$

$$\int 3x^2 \sec^2(x^3) dx = \int 3x^2 \sec^2(u) \cdot \frac{1}{3x^2} du = \int \sec^2 u du = \tan u + C = \tan(x^3) + C$$

Check:

$$\frac{d}{dx} [\tan(x^3)] = 3x^2 \sec^2(x^3)$$

5. Use $u = 5x \rightarrow du = 5 dx \rightarrow \frac{1}{5} du = dx$

$$\int \csc^2(5x) dx = \int \csc^2(u) \left(\frac{1}{5}\right) du = \int \frac{1}{5} \csc^2 u du = -\frac{1}{5} \cot u + C = -\frac{1}{5} \cot(5x) + C$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{5} \cot(5x) \right] = -\frac{1}{5} (-\csc^2(5x)) (5) = \csc^2(5x)$$

6. Use $u = 3x - 5 \rightarrow du = 3 dx \rightarrow \frac{1}{3} du = dx$

$$\int e^{3x-5} dx = \int \frac{1}{3} e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x-5} + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{3} e^{3x-5} \right] = \frac{1}{3} e^{3x-5} (3) = e^{3x-5}$$

7. Use $u = x^3 \rightarrow du = 3x^2 dx \rightarrow \frac{1}{3x^2} du = dx$

$$\int 2x^2 e^{x^3} dx = \int 2x^2 e^u \left(\frac{1}{3x^2}\right) du = \int \frac{2}{3} e^u du = \frac{2}{3} e^u + C = \frac{2}{3} e^{x^3} + C$$

Check:

$$\frac{d}{dx} \left[\frac{2}{3} e^{x^3} \right] = \frac{2}{3} e^{x^3} (3x^2) = 2x^2 e^{x^3}$$

8. Use $u = x^2 + 2x + 1 \rightarrow du = (2x + 2) dx \rightarrow \frac{1}{2(x+1)} du = dx$

$$\int (x+1) e^{x^2+2x+1} dx = \int (x+1) e^u \left(\frac{1}{2(x+1)}\right) du = \int \frac{1}{2} e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2+2x+1} + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{2} e^{x^2+2x+1} \right] = \frac{1}{2} e^{x^2+2x+1} (2x+2) = (x+1) e^{x^2+2x+1}$$

9. Use $u = \sin x \rightarrow du = \cos x \, dx \rightarrow \frac{1}{\cos x} \, du = dx$

$$\int 4 \cos x e^{\sin x} \, dx = \int 4 \cos x e^u \left(\frac{1}{\cos x} \right) \, du = \int 4 e^u \, du = 4 e^u + C = 4 e^{\sin x} + C$$

Check:

$$\frac{d}{dx} [4 e^{\sin x}] = 4 \cos x e^{\sin x}$$

10. Use $u = \tan x \rightarrow du = \sec^2 x \, dx \rightarrow \frac{1}{\sec^2 x} \, du = dx$

$$\int 2 \sec^2 x e^{\tan x} \, dx = \int 2 \sec^2 x e^u \left(\frac{1}{\sec^2 x} \right) \, du = \int 2 e^u \, du = 2 e^u + C = 2 e^{\tan x} + C$$

Check:

$$\frac{d}{dx} [2 e^{\tan x}] = 2 \sec^2 x e^{\tan x}$$