

Antiderivatives 5

Find the indicated antiderivative. Check your answers.

1. Use $u = x^2 + 1 \rightarrow du = 2x dx \rightarrow \frac{1}{2x} du = dx$

$$\int \frac{2x}{x^2 + 1} dx = \int \frac{2x}{u} \cdot \frac{1}{2x} du = \int \frac{1}{u} du = \ln |u| + C = \ln(x^2 + 1) + C$$

Check:

$$\frac{d}{dx} [\ln(x^2 + 1)] = \frac{\frac{d}{dx} [x^2 + 1]}{x^2 + 1} = \frac{2x}{x^2 + 1}$$

2. Use $u = 1 + \sin x \rightarrow du = \cos x dx \rightarrow \frac{1}{\cos x} du = dx$

$$\int \frac{\cos x}{1 + \sin x} dx = \frac{\cos x}{u} \cdot \frac{1}{\cos x} du = \int \frac{1}{u} du = \ln |u| + C = \ln |1 + \sin x| + C$$

Check:

$$\frac{d}{dx} [\ln |1 + \sin x|] = \frac{\frac{d}{dx} [1 + \sin x]}{1 + \sin x} = \frac{\cos x}{1 + \sin x}$$

3. Use $u = 1 + x^4 \rightarrow du = 4x^3 dx \rightarrow \frac{1}{4x^3} du = dx$

$$\int \frac{x^3}{1 + x^4} dx = \int \frac{x^3}{u} \cdot \frac{1}{4x^3} du = \int \frac{1}{4} \left(\frac{1}{u} \right) du = \frac{1}{4} \ln |u| + C = \frac{1}{4} \ln(1 + x^4) + C$$

Check:

$$\frac{d}{dx} \left[\frac{1}{4} \ln(1 + x^4) \right] = \left(\frac{1}{4} \right) \frac{\frac{d}{dx} [1 + x^4]}{1 + x^4} = \frac{1}{4} \cdot \frac{4x^3}{1 + x^4} = \frac{x^3}{1 + x^4}$$

4. Use $u = e^x + 2x + 7 \rightarrow du = (e^x + 2) dx \rightarrow \frac{1}{e^x + 2} du = dx$

$$\int \frac{e^x + 2}{e^x + 2x + 7} dx = \int \frac{e^x + 2}{u} \cdot \frac{1}{e^x + 2} du = \int \frac{1}{u} du = \ln |u| + C = \ln |e^x + 2x + 7| + C$$

Check:

$$\frac{d}{dx} [\ln |e^x + 2x + 7|] = \frac{\frac{d}{dx} [e^x + 2x + 7]}{e^x + 2x + 7} = \frac{e^x + 2}{e^x + 2x + 7}$$

5. Use $u = 2x + 5 \rightarrow du = 2 dx \rightarrow \frac{1}{2} du = dx$

$$\int \frac{3}{2x + 5} dx = \int \frac{3}{u} \cdot \frac{1}{2} du = \int \frac{3}{2} \left(\frac{1}{u} \right) du = \frac{3}{2} \ln |u| + C = \frac{3}{2} \ln |2x + 5| + C$$

Check:

$$\frac{d}{dx} \left[\frac{3}{2} \ln |2x + 5| \right] = \frac{3}{2} \cdot \frac{\frac{d}{dx} [2x + 6]}{(2x + 5)} = \frac{3}{2} \cdot \frac{2}{(2x + 5)} = \frac{3}{2x + 5}$$

6. Use $u = x^2 + 1 \rightarrow du = 2x dx \rightarrow \frac{1}{2x} du = dx$

$$\int \frac{2x}{(x^2 + 1)^2} dx = \int \frac{2x}{(u)^2} \cdot \frac{1}{2x} du = \int u^{-2} du = -u^{-1} + C = -\frac{1}{x^2 + 1} + C$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{x^2 + 1} \right] = \frac{d}{dx} \left[-(x^2 + 1)^{-1} \right] = -(-1)(x^2 + 1)^{-2} (2x) = \frac{2x}{(x^2 + 1)^2}$$

7. Use $u = 1 + \sin x \rightarrow du = \cos x dx \rightarrow \frac{1}{\cos x} du = dx$

$$\int \frac{\cos x}{(1 + \sin x)^2} dx = \int \frac{\cos x}{(u)^2} \frac{1}{\cos x} du = \int u^{-2} du = -u^{-1} + C = -\frac{1}{1 + \sin x} + C$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{1 + \sin x} \right] = \frac{d}{dx} \left[-(1 + \sin x)^{-1} \right] = -(-1)(1 + \sin x)^{-2} (\cos x) = \frac{\cos x}{(1 + \sin x)^2}$$

8. Use $u = 1 + x^4 \rightarrow du = 4x^3 dx \rightarrow \frac{1}{4x^3} du = dx$

$$\int \frac{x^3}{(1 + x^4)^3} dx = \int \frac{x^3}{(u)^3} \cdot \frac{1}{4x^3} du = \int \frac{1}{4} u^{-3} du = \frac{1}{4} \left(-\frac{1}{2} \right) u^{-2} + C = -\frac{1}{8(1 + x^4)^2} + C$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{8(1 + x^4)^2} \right] = \frac{d}{dx} \left[-\frac{1}{8} (1 + x^4)^{-2} \right] = -\frac{1}{8} (-2) (1 + x^4)^{-3} (4x^3) = \frac{x^3}{(1 + x^4)^3}$$

9. Use $u = e^x + 2x + 7 \rightarrow du = (e^x + 2) dx \rightarrow \frac{1}{e^x + 2} du = dx$

$$\begin{aligned}\int \frac{e^x + 2}{(e^x + 2x + 7)^3} dx &= \int \frac{(e^x + 2)}{(u)^3} \cdot \frac{1}{(e^x + 2)} du = \int u^{-3} du \\ &= -\frac{1}{2}u^{-2} + C = -\frac{1}{2(e^x + 2x + 7)^2} + C\end{aligned}$$

Check:

$$\begin{aligned}\frac{d}{dx} \left[-\frac{1}{2(e^x + 2x + 7)^2} \right] &= \frac{d}{dx} \left[-\frac{1}{2} (e^x + 2x + 7)^{-2} \right] \\ &= -\frac{1}{2}(-2)(e^x + 2x + 7)^{-3}(e^x + 2) = \frac{e^x + 2}{(e^x + 2x + 7)^3}\end{aligned}$$

10. Use $u = 2x + 5 \rightarrow du = 2 dx \rightarrow \frac{1}{2} du = dx$

$$\int \frac{3}{(2x + 5)^4} dx = \int \frac{3}{(u)^4} \left(\frac{1}{2} \right) du = \int \frac{3}{2} u^{-3} du = \frac{3}{2} \left(-\frac{1}{3} \right) u^{-2} + C = -\frac{1}{2(2x + 5)^3} + C$$

Check:

$$\frac{d}{dx} \left[-\frac{1}{2(2x + 5)^3} \right] = \frac{d}{dx} \left[-\frac{1}{2}(2x + 5)^{-3} \right] = -\frac{1}{2}(-3)(2x + 5)^{-4}(2) = \frac{3}{(2x + 5)^4}$$