March 11, 2020Name

Antiderivatives 6

Find the indicated antiderivative. Check your answers.

1. Identity

$$\int \frac{5}{\sqrt{1-x^2}}\,dx = 5\int \frac{1}{\sqrt{1-x^2}}\,dx = 5\sin^{-1}x + C$$

Check:

$$\frac{d}{dx}\left[5\sin^{-1}x\right] = 5\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{5}{\sqrt{1-x^2}}$$

2. Use $u = 1-x^2 \to du = -2x\,dx \to -\frac{1}{2x}\,du = dx$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{u}}\left(-\frac{1}{2x}\right)\,du = \int -\frac{1}{2}u^{-1/2}\,du = -\frac{1}{2}(2)u^{1/2}+C = -\sqrt{1-x^2}+C$$

Check:

$$\frac{d}{dx}\left[-\sqrt{1-x^2}\right] = -\frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}$$

3. Use $u = 1-x^4 \to du = -4x^3\,dx \to -\frac{1}{4x^3}\,du = dx$

$$\int \frac{4x^3}{\sqrt{1-x^4}}\,dx = \int \frac{4x^3}{\sqrt{u}}\left(-\frac{1}{4x^3}\right)\,du = \int -u^{-1/2}\,du = -2u^{1/2}+C = -2\sqrt{1-x^4}+C$$

Check:

$$\frac{d}{dx}\left[-2\sqrt{1-x^4}\right] = -2\left(\frac{1}{2}\right)\left(1-x^4\right)^{-1/2}\left(-4x^3\right) = \frac{4x^3}{\sqrt{1-x^4}}$$

4. Use $u = x^2 \to du = 2x\,dx \to \frac{1}{2x}\,du = dx$

$$\int \frac{4x}{\sqrt{1-x^4}}\,dx = \int \frac{4x}{\sqrt{1-u^2}}\cdot\frac{1}{2x}\,du = \int 2\left(\frac{1}{\sqrt{1-u^2}}\right)\,du = 2\sin^{-1}u+C = 2\sin^{-1}\left(x^2\right)+C$$

Check:

$$\frac{d}{dx}\left[2\sin^{-1}\left(x^2\right)\right] = 2\left(\frac{1}{\sqrt{1-\left(x^2\right)^2}}\right)(2x) = \frac{4x}{\sqrt{1-x^4}}$$

5. Use $u = 1 - x^4 \rightarrow du = -4x^3\,dx \rightarrow -\frac{1}{4x^3}\,du = dx$

$$\int x^3\sqrt{1-x^4}\,dx = \int x^3\sqrt{u}\left(-\frac{1}{4x^3}\right)du = \int -\frac{1}{4}\sqrt{u}\,du$$

$$= -\frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{6}\left(1-x^4\right)^{3/2} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{6}\left(1-x^4\right)^{3/2}\right] = -\frac{1}{6}\cdot\frac{3}{2}\left(1-x^4\right)^{1/2}\left(-4x^3\right) = x63\sqrt{1-x^4}$$

6. Use $u = 1 + x^4 \rightarrow du = 4x^3\,dx \rightarrow \frac{1}{4x^3}\,du = dx$

$$\int \frac{2x^3}{1+x^4}\,dx = \int \frac{2x^3}{u}\cdot\frac{1}{4x^3}\,du = \int \frac{1}{2}\left(\frac{1}{u}\right)du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left(1+x^4\right) + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{2}\ln\left(1+x^4\right)\right] = \frac{1}{2}\left(\frac{4x^3}{1+x^4}\right) = \frac{2x^3}{1+x^4}$$

7. Use $u = x^2 \rightarrow du = 2x\,dx \rightarrow \frac{1}{2x}\,du = dx$

$$\int \frac{2x}{1+x^4}\,dx = \int \frac{2x}{1+u^2}\cdot\frac{1}{2x}\,du = \int \frac{1}{1+u^2}\,du = \tan^{-1}u + C = \tan^{-1}\left(x^2\right) + C$$

Check:

$$\frac{d}{dx}\left[\tan^{-1}\left(x^2\right)\right] = \frac{1}{1+\left(x^2\right)^2}(2x) = \frac{2x}{1+x^4}$$

8. Use $u = x^6 - 1 \rightarrow du = 6x^5\,dx \rightarrow \frac{1}{6x^5}\,du = dx$

$$\int \frac{6x^5}{\sqrt{x^2-1}}\,dx = \int \frac{6x^5}{\sqrt{u}}\cdot\frac{1}{lx^5}\,du = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{x^6-1} + C$$

Check:

$$\frac{d}{dx}\left[2\sqrt{x^6-1}\right] = 2\left(\frac{1}{2}\right)\left(x^6-1\right)^{-1/2}\left(6x^5\right) = \frac{6x^5}{\sqrt{x^6-1}}$$

9. Use $u = x^3 \to du = 3x^2\,dx \to \frac{1}{3x^2}\,du = dx$

$$\int \frac{6}{x\sqrt{x^6-1}}\,dx = \int \frac{6}{x\sqrt{u^2-1}} \cdot \frac{1}{3x^2}\,du = \int \frac{2}{x^3\sqrt{u^2-}}\,du = 2\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= 2\sec^{-1} u + C = 2\sec^{-1}\left(x^3\right) + C$$

$$\frac{d}{dx}\left[2\sec^{-1}\left(x^3\right)\right] = 2\left(\frac{1}{x^3\sqrt{\left(x^3\right)^2-1}}\right)\left(3x^2\right) = \frac{6x^2}{x^3\sqrt{x^6-1}} = \frac{6}{x\sqrt{x^6-1}}$$

10. Use $u = x^6 - 1 \to du = 6x^5\,dx \to \frac{1}{6x^5}\,du = dx$

$$\int 6x^5\sqrt{x^6-1}\,dx = \int 6x^5\sqrt{u}\left(\frac{1}{6x^5}\right)\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2}+C = \frac{2}{3}\left(x^6-1\right)^{3/2}+C$$

Check:

$$\frac{d}{dx}\left[\frac{2}{3}\left(x^6-1\right)^{3/2}\right] = \frac{2}{3}\cdot\frac{3}{2}\left(x^6-1\right)^{1/2}\left(6x^5\right) = 6x^5\sqrt{x^6-1}$$