March 11, 2020Name

Antiderivatives 7

Find the indicated antiderivative. Check your answers.
1. Use $u \sin x \to du = \cos x\, dx$

$$\int \sin^3 x \cos x\, dx = \int u^5\, du = \frac{1}{6}u^6 + C = \frac{1}{6}\sin^6 x + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{6}\sin^6 x\right] = \frac{1}{6}(6)\sin^5 x \cos x = \sin^5 x \cos x$$

2. Use $u = \cos x \to du = -\sin x\, dx \to -1\, du = \sin x\, dx$

$$int \cos^4 x \sin x\, dx = \int u^4(-1)\, du = -\frac{1}{5}u^5 + C = -\frac{1}{5}\cos^5 x + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{5}\cos^5 x\right] = -\frac{1}{5}(5)\cos^4 x(-\sin x) = \cos^4 x \sin x$$

3. Use $u = \tan x \to du = \sec^2 x\, dx$

$$\int \tan^5 x \sec^2 x\, dx = \int u^5\, du = \frac{1}{6}u^6 + C = \frac{1}{6}\tan^6 x + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{6}\tan^6 x\right] = \frac{1}{6}(6)\tan^5 x \sec^2 x = \tan^5 x \sec^2 x$$

4. Use $u = \tan x \to du = \sec^2 x\, dx$

$$\int \tan^2 x \sec^2 x\, dx = \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}\tan^3 x + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{3}\tan^3 x\right] = \frac{1}{3}(3)\tan^3 x \sec^2 x = \tan^2 x \sec^2 x$$

5. Use $u = \cot x \to du = -\csc^2 x\, dx \to -1\, du = \csc^2 x\, dx$

$$\int \cot^3 x \csc^2 x\, dx = \int u^3(-1)\, du = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cot^4 x + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{4}\cot^4 x\right] = -\frac{1}{4}(4)\cot^3 x\left(-\csc^2 x\right) = \cot^3 x \csc^2 x$$

Use $u = x + 5 \to du = dx$

$$\int \frac{1}{x+5}\, dx = \int \frac{1}{u}\, du = \ln|u| + C = \ln|x+5| + C$$

Check:

$$\frac{d}{dx}\left[\ln|x+5|\right] \frac{\frac{d}{dx}[x+5]}{x+5} = \frac{1}{x+5}$$

7. Use $u = x + 3 \to du = dx \& u - 3 = x$

$$\int \frac{x}{x+3}\, dx = \int \frac{u-3}{u}\, du = \int 1 - \frac{3}{u}\, du = u - 3\ln|u| + C = x + 3 - 3\ln|x+3| + C$$

Check:

$$\frac{d}{dx}\left[x + 3 - 3\ln|x+3|\right] = 1 - 3\left(\frac{1}{x+3}\right) = \frac{x+3}{x+3} - \frac{3}{x+3} = \frac{x}{x+3}$$

8. Use $u = x + 2 \to du = dx$

$$\int \frac{1}{\sqrt{x+2}}\, dx = \int \frac{1}{\sqrt{u}}\, du = \int u^{-1/2}\, du = 2u^{1/2} + C = 2\sqrt{x+2} + C$$

Check:

$$\frac{d}{dx}\left[2\sqrt{x+2}\right] = 2\left(\frac{1}{2}\right)(x+2)^{-1/2}(1) = \frac{1}{\sqrt{x+2}}$$

9. Use $u = x + 2 \to du = dx \& x = u - 2$

$$\int \frac{x}{\sqrt{x+2}}\, dx = \int \frac{u-2}{\sqrt{u}}\, du = \int (u-2)u^{-1/2}\, du$$

$$= \int u^{1/2} - 2u^{-1/2}\, du = \frac{2}{3}u^{3/2} - 2(2)u^{1/2} + C = \frac{2}{3}(x+2)^{3/2} - 4(x+2)^{1/2} + C$$

Check:

$$\frac{d}{dx}\left[\frac{2}{3}(x+2)^{3/2} - 4(x+2)^{1/2}\right] = \frac{2}{3} \cdot \frac{3}{2}(x+2)^{1/2} - 4\left(\frac{1}{2}\right)(x+2)^{-1/2}$$

$$= \sqrt{x+2} - \frac{2}{\sqrt{x+2}} = \sqrt{x+2}\left(\frac{\sqrt{x+2}}{\sqrt{x+2}}\right) - \frac{2}{\sqrt{x+2}} = \frac{x+2}{\sqrt{x+2}} - \frac{2}{\sqrt{x+2}} = \frac{x}{\sqrt{x+2}}$$

10. Use $u = x + 2 \rightarrow du = dx \& x = u - 2$

$$\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du = \int u^{3/2} - 2u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} - 2\left(\frac{2}{3}\right)u^{3/2} + C = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

Check:

$$\frac{d}{dx}\left[\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2}\right] = \frac{2}{5}\cdot\frac{5}{2}(x+2)^{3/2} - \frac{4}{3}\cdot\frac{3}{2}(x+2)^{1/2}$$

$$= (x+2)^{3/2} - 2(x+2)^{1/2} = (x+2)\sqrt{x+2} - 2\sqrt{x+2} = x\sqrt{x+2}$$