March 11, 2020Name

Antiderivatives 8

Find the indicated antiderivative. Check your answers.

1. Use $u = \ln x \to du = \frac{1}{x}\,dx \to x\,du = dx$

$$\int \frac{(\ln x)^2}{x}\,dx = \int \frac{(u)^2}{x}(x)\,du = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{3}(\ln x)^3\right] = \frac{1}{3}(3)(\ln x)^2\left(\frac{1}{x}\right) = \frac{(\ln x)^2}{x}$$

2. Use $u = \ln x \to du = \frac{1}{x}\,dx \to x\,du = dx$

$$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{xu}(x)\,du = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$$

Check:

$$\frac{d}{dx}\left[\ln|\ln x|\right] = \frac{\frac{d}{dx}[\ln x]}{\ln x} = \frac{\frac{1}{x}}{\frac{\ln x}{1}} = \frac{1}{x}\cdot\frac{1}{\ln x} = \frac{1}{x\ln x}$$

3. Use $u = \sec^{-1} x \to du = \frac{1}{x\sqrt{x^2-1}}\,dx \to x\sqrt{x^2-1}\,du = dx$

$$\int \frac{e^{\sec^{-1} x}}{x\sqrt{x^2-1}}\,dx = \int \frac{e^u}{x\sqrt{x^2-1}}\left(x\sqrt{x^2-1}\right)du = \int e^u\,du = e^u + C = e^{\sec^{-1} x} + C$$

Check:

$$\frac{d}{dx}\left[e^{\sec^{-1} x}\right] = e^{\sec^{-1} x}\left(\frac{1}{x\sqrt{x^2-1}}\right) = \frac{e^{\sec^{-1} x}}{x\sqrt{x^2-1}}$$

4. Split into two integrals & use $u = x^2 + 1$ for the first one . . .

$$\int \frac{2x+1}{x^2+1}\,dx = \int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = \ln\left(x^2+1\right) + \tan^{-1} x + C$$

Check:

$$\frac{d}{dx}\left[\ln\left(x^2+1\right) + \tan^{-1} x\right] = \frac{\frac{d}{dx}\left[x^2+1\right]}{x^2+1} + \frac{1}{x^2+1} = \frac{2x}{x^2+1} + \frac{1}{x^2+1} = \frac{2x+1}{x^2+1}$$

5. Use $u = \tan^{-1} x \to du = \frac{1}{x^2+1}\,dx \to \left(x^2+1\right)du = dx$

$$\int \frac{\tan^{-1} x}{x^2+1}\,dx = \int \frac{u}{x^2+1}\left(x^2+1\right)du = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\left(\tan^{-1} x\right)^2 + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{2}\left(\tan^{-1}x\right)^2\right] = \frac{1}{2}(2)\left(\tan^{-1}x\right)\left(\frac{1}{x^2+1}\right) = \frac{\tan^{-1}x}{x^2+1}$$

6. Use $u = \ln x \to du = \frac{1}{x}\,dx \to x\,du = dx$

$$\int \frac{1}{x(\ln x)^2}\,dx = \int \frac{1}{x(u)^2}(x)\,du = \int u^{-2}\,du = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{\ln x}\right] = \frac{d}{dx}\left[-\left(\ln x\right)^{-1}\right] = -(-1)\left(\ln x\right)^{-2}\left(\frac{1}{x}\right) = \frac{1}{x\left(\ln x\right)^2}$$

7. Use $u = \sin^{-1}x \to du = \frac{1}{\sqrt{1-x^2}}\left(\sqrt{1-x^2}\right)du = dx$

$$\int \frac{1}{\left(\sin^{-1}x\right)^3\sqrt{1-x^2}}\,dx = \int \frac{1}{(u)\sqrt{1-x^2}}\left(\sqrt{1-x^2}\right)du$$

$$= \int u^{-3}\,du = -\frac{1}{2}u^{-2} + C = -\frac{1}{2\left(\sin^{-1}x\right)^2} + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{2\left(\sin^{-1}x\right)^2}\right] = \frac{d}{dx}\left[-\frac{1}{2}\left(\sin^{-1}x\right)^{-2}\right] = -\frac{1}{2}(-2)\left(\sin^{-1}x\right)^{-3}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$= \frac{1}{\left(\sin^{-1}x\right)^3\sqrt{1-x^2}}$$

8. Use $u = \tan^{-1}x \to du = \frac{1}{x^2+1}\,dx \to \left(x^2+1\right)du = dx$

$$\int \frac{\left(\tan^{-1}x\right)^2}{x^2+1}\,dx = \int \frac{(u)^2}{x^3+1}\left(x^2+1\right)du = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\left(\tan^{-1}x\right)^3 + C$$

Check:

$$\frac{d}{dx}\left[\frac{1}{3}\left(\tan^{-1}x\right)^3\right] = \frac{1}{3}(3)\left(\tan^{-1}x\right)^2\left(\frac{1}{x^2+1}\right) = \frac{\left(\tan^{-1}x\right)^2}{x^2+1}$$

9. Use $u = 1 - e^{2x} \to du = -2e^{1x}\,dx \to -\frac{1}{2e^{2x}}\,du = dx$

$$\int \frac{e^{2x}}{1-e^{2x}}\,dx = \int \frac{e^{2x}}{u}\left(-\frac{1}{2e^{2x}}\right)du$$

$$= \int -\frac{1}{2}\left(\frac{1}{u}\right)du = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln\left|1-e^{2x}\right| + C$$

Check:

$$\frac{d}{dx}\left[-\frac{1}{2}\ln\left|1-e^{2x}\right|\right] = -\frac{1}{2}\left(\frac{\frac{d}{dx}\left[1-e^{2x}\right]}{1-e^{2x}}\right) = -\frac{1}{2}\cdot\frac{-2e^{2x}}{(1-e^{2x})} = \frac{e^{2x}}{1-e^{1x}}$$

10. Use $u = e^x \to du = e^x\,dx \to \frac{1}{e^x}\,du = dx$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx = \int \frac{e^x}{\sqrt{1-u^2}}\cdot\frac{1}{e^x}\,du$$

$$= \int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}u + C = \sin^{-1}(e^x) + C$$

Check:

$$\frac{d}{dx}\left[\sin^{-1}(e^x)\right] = \frac{1}{\sqrt{1-(e^x)^2}}(e^x) = \frac{e^x}{\sqrt{1-e^{2x}}}$$