Friday - Week 1

Warm-up:

1. Find an equation of the line perpendicular to the segment connecting (1,2) and (6,10) which goes through the midpoint. This is the perpendicular bisector.

1. Verify your answer in Desmos with a graph.

Recall the midpoint of a segment with endpoints (x_1, y_1) & (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Slope of the segment: $\frac{10-2}{6-1} = \frac{8}{5}$

Slope of the $\perp$: $\frac{-5}{8}$

Midpoint: $\left(\frac{1+6}{2}, \frac{2+10}{2}\right) = \left(\frac{7}{2}, \frac{12}{2}\right)$

Eq'n of $\perp$ bisector: $y - 6 = \frac{-5}{8}\left(x - \frac{7}{2}\right)$

Continue Review of the Basics

0. Rationalizing
1. functions
   (a) domain
   (b) composition/evaluation
   (c) even/odd

Typical Rationalizing Problem:

$$\frac{abc}{a} = bc$$

Rationalizing the numerator.

$$\frac{\sqrt{x}+\sqrt{y}}{xy}.$$

$$\frac{\left(\sqrt{x}+\sqrt{y}\right)}{xy}\cdot\frac{\left(\sqrt{x}-\sqrt{y}\right)}{\sqrt{x}-\sqrt{y}} = \frac{\overbrace{\sqrt{x}\sqrt{x}}^{=x}\ \overbrace{-\sqrt{x}\sqrt{y}+\sqrt{y}\sqrt{x}}^{=0}\ -\overbrace{\sqrt{y}\sqrt{y}}^{=y}}{xy\left(\sqrt{x}-\sqrt{y}\right)} = \frac{x-y}{xy\left(\sqrt{x}-\sqrt{y}\right)}$$

cancellation across denominators doesn't work well with + or -

And speaking of cancellation ...

Simplify (if possible)

$$\frac{(x+y)^2}{(xy)(x^2+y^2)} = \frac{x^2+2xy+y^2}{(x^3y+xy^3)} = \frac{2xy+x^2+y^2}{x^3y+xy^3}$$

Domain of functions

Can't divide by 0.

Def'n: $\frac{a}{b} = c$ means $a = cb$

$\frac{5}{0} = c$ then $5 = \underbrace{c \cdot 0}_{=0}$

$5 = 0$ ☹

Can't take the square root of negative.

$\sqrt{x} = n$ & $n \cdot n = x$

when you multiply a number by itself, the result is always positive

Ex

$f(x) = \frac{1}{x^2 - 25}$

domain: $\mathbb{R} - \{\pm 5\} = (-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

think: can't $\div$ by 0

set $x^2 - 25 = 0$

$(x-5)(x+5) = 0$

$x - 5 = 0 \quad x = 5$

or

$x + 5 = 0 \quad x = -5$

Ex $f(x) = \sqrt{3x+9}$

domain:

think radicand must be 0 or greater.

$3x + 9 \geq 0$

$3x \geq -9$

$x \geq -3$

$[-3, \infty)$

$g(x) = \sqrt{x^2 - 16}$

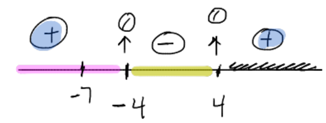
domain:

$x^2 - 16 \geq 0$

① set $x^2 - 16 = 0$

$x = \pm 4$

②



$(-\infty, -4] \cup [4, \infty)$