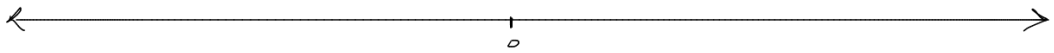
Real Numbers



Exponents:

$5^3 = 5 \cdot 5 \cdot 5 = 125$

$(2^3)^4 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 = 2^{12}$ raise a power to a power -> multiply powers

$(3x)^4 = 3x \cdot 3x \cdot 3x \cdot 3x = 3^4 \cdot x^4$ BECAUSE of MULTIPLICATION, you apply the 4 to both the 3 and x

commutative

$a^b \cdot c^b = (ac)^b$

$(3+x)^4 \neq 3^4 + x^4$ exponents do not play nicely with + and -

Binomial Expansion

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Ex:

$(3x+5)^3 = (3x+5)^2(3x+5)$

$= (9x^2 + 30x + 25)(3x+5)$

$= (9x^2 + 30x + 25)3x + (9x^2 + 30x + 25) \cdot 5$

$= 27x^3 + 90x^2 + 75x + 45x^2 + 150x + 125$

$= 27x^3 + 135x^2 + 225x + 125$

$(3x)^3 + 3(3x)^2 \cdot 5 + 3(3x) \cdot 5^2 + 125$

Absolute value

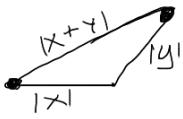
Def: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

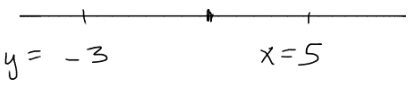
Properties:

$|x \cdot y| = |x||y|$ ($|-3 \cdot 5| = |-15| = 15 = |-3||5|$

$|x + y| \leq |x| + |y|$

Triangle inequality





$|-3+5| = 2$

Set Notation -

$x \in \mathbb{R}$

"in"

x lives in the set of real numbers

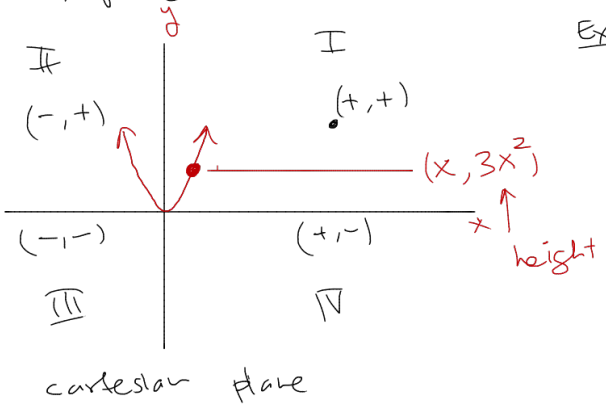
Ex

$x \in [5, 10)$

Ex

$[1,3] \cup [5,7)$

Graphing

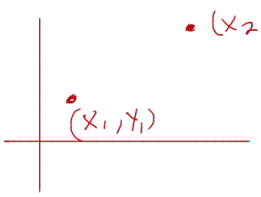


cartesian plane

Ex $y = 3x^2$ — equation

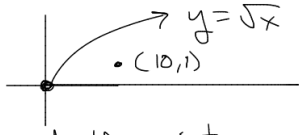
— cuts out "distinguishes" a curve on the plane where every point on curve is (x, y) $\|$ $(x, 3x^2)$

(Euclidean) Distance Formula



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Later, we'll solve

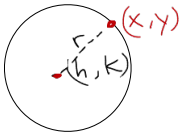


Find the point on curve closest to (10,1)

distance from (10,1) to various pts

Circle Formula

A circle is the set of points equidistant to a single point (its center)



$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Functions

Input / Output Machines

$f(x) = 3x + 1$ (linear function)

output

f of x

input goes here

Ex

$f(5) = 3 \cdot 5 + 1$

$f(a+h) = 3(a+h) + 1 = 3a + 3h + 1$

$f(x+h) = 3x + 3h + 1$

Ex $f(x) = 5x^2 + 3x + 1$

$f(x+h) = 5(x+h)^2 + 3(x+h) + 1 = 5(x^2 + 2xh + h^2) + 3x + 3h + 1$

$= 5x^2 + 10xh + 5h^2 + 3x + 3h + 1$

$f(x+h) - f(x) =$

$5x^2 + 10xh + 5h^2 + 3x + 3h + 1 - (5x^2 + 3x + 1)$

don't forget parenthesis here

$10xh + 5h^2 + 3h$