MA161 – wk 1 Wed

FROM YESTERDAY

Difference Quotient Simplifications

$f(x) = \frac{1}{x}$

compute: $\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x}{x}\left(\frac{1}{x+h}\right) - \frac{x+h}{x+h}\left(\frac{1}{x}\right)}{h}$

$f(5) = \frac{1}{5}$

$f(2+3) = \frac{1}{2+3}$

$= \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\left[\frac{-h}{x(x+h)}\right]}{h} = \frac{-h}{x(x+h)} \cdot \frac{1}{h}$

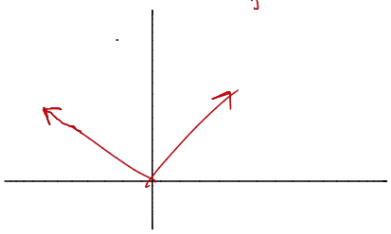
$\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}$

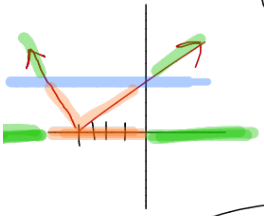
$= \frac{-1}{x(x+h)}$

Abs. Value Inequalities

$|x + 4| < 10$

think both sides are y-values (heights) of graph





less than ⇒ "and"

$-10 < x + 4 < 10$ solve simultaneously

VS

$|x + 4| > 10$

greater than ⇒ "or"

$x + 4 > 10$ or $x + 4 < -10$

Functions

Domain: set of allowable inputs

↳ 1. can't ÷ by 0 (why? $\frac{a}{b} = c$ means $a = b \cdot c$ so if $\frac{5}{0} = x$ then $5 = x \cdot 0 = 0$)

2. can't square-root a negative

3. can't log a negative

Range: set of achievable outputs

Transformations:

given $f(x)$, if

$h(x) = k \cdot f(x)$

$h(x) = f(kx)$

$h(x) = f(x) + k$

$h(x) = f(x+k)$

$k \in \mathbb{R}$

tranny's affect y-values

x-values are changed

EX $f(x) = \sin(x)$, restrict domain to $[\frac{\pi}{2}, 2\pi]$

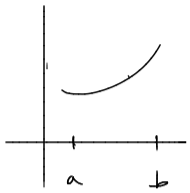
$g(x) = f(5x)$.

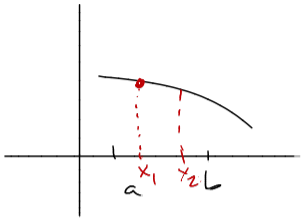
Domain of f is $[\frac{\pi}{2}, 2\pi]$

Domain of $g(x)$? Set $5x \in [\frac{\pi}{2}, 2\pi] \Rightarrow x \in [\frac{\pi}{10}, \frac{2\pi}{5}]$

Range of $g(x)$? Same as range of $f(x)$

Increasing / Decreasing





$f(x)$ increases on (a,b) if when $x_1 < x_2$ we have $f(x_1) < f(x_2)$

"preserved"

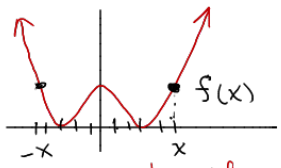
for all $x_i \in (a,b)$!

$f(x)$ decreases on (a,b) if when $x_1 < x_2$ we get $f(x_1) > f(x_2)$

Even & Odd Functions

Gist

Even



symmetry about the y-axis

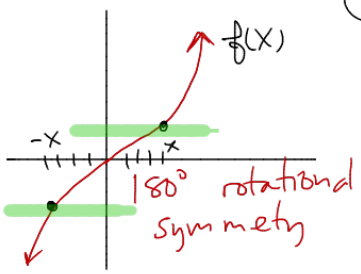
(Ex) $y = \cos x$ is even

$\cos(190^\circ) = \cos(-190^\circ)$

Def'n

$f(-x) = f(x)$

odd



(Ex) $y = \sin(x)$ is odd

$\sin(-\pi/3) = -\sin(\pi/3)$

$f(-x) = -f(x)$

Line

$y - y_1 = m(x - x_1)$ point-slope

$y = mx + b$ slope-int

slope $m = \dfrac{y - y_1}{x - x_1}$

parallel $\parallel$ same slope

perpendicular $\perp$ $\Rightarrow$ negative reciprocal

Ex. Find an eq'n of the line between (3,5) and (7,−2)

need (two ingredients)

$$m = \frac{5 - (-2)}{3 - 7} = \frac{7}{-4}$$

$y - 5 = \frac{7}{-4}(x - 3)$ (or) $y + 2 = \frac{7}{-4}(x - 7)$

Common Calculus Simplification

$f(x) = \sqrt{x}$

$f(16) = \sqrt{16}$

$f(x+h) = \sqrt{x+h}$

$$\frac{f(x+h) - f(x)}{h} =$$

(difference quotient)

$$\frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$