MA161 Wk 10

warm-up

Find $f(x)$ if $f''(x) = \cos(x)$, $f'(0) = 1$, $f(\pi/2) = 2$

$f'(x) = \int f''(x)dx = \int \cos(x)dx = \sin(x) + C$

$f'(0) = 1 = \underbrace{\sin(0)}_{=0} + c = c \Rightarrow \boxed{f'(x) = \sin(x) + 1}$

Repeat:

$f(x) = \int f'(x)dx = \int \sin(x) + 1\, dx = -\cos(x) + x + C$

$f(\frac{\pi}{2}) = -\underbrace{\cos(\frac{\pi}{2})}_{=0} + \frac{\pi}{2} + C = 2 \Rightarrow C = 2 - \frac{\pi}{2}$

$\boxed{f(x) = -\cos(x) + x + 2 - \frac{\pi}{2}}$

Follow-up: Suppose a particle is accelerating according to $a(t) = \cos(t)$ where t measured in seconds & the units of $a(t)$ are ft/s^2. Suppose the initial velocity is 1 ft/sec. Suppose @ $t = \pi/2$ (≈ 1.5 seconds) the particle is 2 ft high. Find a formula for the position of the particle and use it to find the height @ time $t = \pi$ (≈ 3.14 seconds)

By work above $s(t) = -\cos(t) + t + 2 - \pi/2$ ft $\Longrightarrow s(\pi) = -\underbrace{\cos(\pi)}_{} + \pi + 2 - \frac{\pi}{2} = 3 + \frac{\pi}{2} \approx 4.5$ ft

1. $\int \frac{5}{\sqrt{1-x^2}}\,dx =$

NO derivative relationships ; (x^2 ———— x

think: inverse trig (see chart) $5x^3+1$ $\overline{\text{degree diffs}}$ x^2

$\sin x$ ———— $\cos x$

e^x ———— e^x

$\int (e^x+1)^5 e^x\,dx$

$u = 1-x^2$

1———— include constants!

2. $\int \frac{x}{\sqrt{1-x^2}}\,dx =$ see: $1-x^2$ ———— x

deg. 1 diff

Inverse trig

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$$

$$\frac{d}{dx}\left(\sin^{-1}(2x)\right) = \frac{1}{\sqrt{1-(2x)^2}}\cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

3. $\int \frac{4x^3}{\sqrt{1-x^4}}\,dx =$

$$\frac{d}{dx}\left(\sin^{-1}(x^2)\right) = \frac{1}{\sqrt{1-(x^2)^2}}\cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

(degree 1 diff.

$u = 1-x^4$

think!

4. $\int \frac{4x}{\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,du$

$x^4 = u^2$

↓ take + square root

No deriv. relationships

sub

set: $\boxed{x^2 = u}$ $\Longrightarrow \frac{d}{dx}(x^2) = \frac{d}{dx}(u)$

$$= \int \frac{4\cancel{x}}{\sqrt{1-u^2}}\,\frac{1}{2\cancel{x}}\,du$$

$$2x = \frac{du}{dx}$$

$$2x\,dx = du$$

$$\boxed{dx = \frac{1}{2x}\cdot du}$$

$$2\int \frac{1}{\sqrt{1-u^2}}\,du = 2\sin^{-1}(u) + C$$

$$= \boxed{2\sin^{-1}(x^2) + C}$$

Challenge

$$\int \frac{e^x}{\sqrt{1+e^x}}\,dx$$

$\boxed{u = 1+e^x}$

$\frac{du}{dx} = e^x$

$du = e^x\,dx$

$\boxed{\frac{1}{e^x}\,du = dx}$

$$= \int \frac{\cancel{e^x}}{\sqrt{u}} \cdot \frac{1}{\cancel{e^x}}\,du$$

$$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du$$

$$= 2u^{\frac{1}{2}} + c$$

$$= \boxed{2(1+e^x)^{\frac{1}{2}} + C}$$

CIE? if so $\Rightarrow \int e^u\,du$

i.e., $\int e^{\sin x} \cdot \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$$= \int e^u\,du$$

$$= e^{\sin x} + c$$

$$\int \frac{1}{e^x\sqrt{e^{2x}-1}}\,dx \overset{\text{think}}{=} \int \frac{1}{u\sqrt{u^2-1}}\,du$$

$e^{2x} = u^2$

$(e^x)^2 = u^2 \xrightarrow{\text{sq. roots}} \boxed{e^x = u}$

$du = e^x\,dx$

$\boxed{\frac{1}{e^x}\,du = dx}$

$\underline{\underline{\text{sub}}}$ $\int \frac{1}{e^x\sqrt{u^2-1}} \cdot \frac{1}{e^x}\,du$

this is what a bad substitution looks like