Wk 10 — Monday

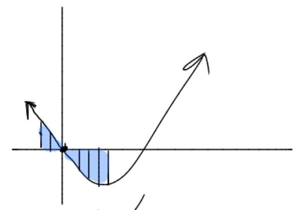
(Ch. 5.1, 5.2, 5.3)

Exercise!

5.1.2. right endpoints w/ $n=5$

Calculate R_5 for $f(x) = 4x^2 + 5x$ over $[-1, 1]$. ($a = -1$, $b = 1$)

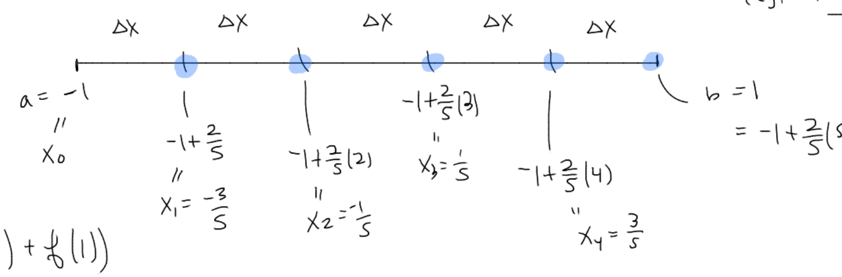
(Give an exact answer. Use symbolic notation and fractions where needed.)



① $n=5$, $\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{5} = \frac{2}{5}$ (width)

$R_5 \Rightarrow$ skip 1st / $L_5 \Rightarrow$ skip last / M_5 = midpts (eg. $\frac{-1+\left(\frac{-3}{5}\right)}{2} = \frac{\frac{-8}{5}}{2} = \frac{-4}{5}$)

② $R_5 = \sum_{i=1}^{5} \Delta x \cdot f(x_i)$



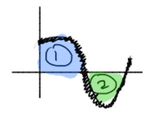
$$= \Delta x \left(f\left(\frac{-3}{5}\right) + f\left(\frac{-1}{5}\right) + f\left(\frac{1}{5}\right) + f\left(\frac{3}{5}\right) + f(1) \right)$$

$$= \frac{2}{5}\left(4\left(\frac{-3}{5}\right)^2 + 5\left(\frac{-3}{5}\right) + 4\left(\frac{-1}{5}\right)^2 + 5\left(\frac{-1}{5}\right) + 4\left(\frac{1}{5}\right)^2 + 5\left(\frac{1}{5}\right) + 4\left(\frac{3}{5}\right)^2 + 5\left(\frac{3}{5}\right) + 4(1)^2 + 5(1)\right)$$

$$= 4.88 = 4 + \frac{88}{100} = \frac{400}{100} + \frac{88}{100} = \frac{488}{100} \xrightarrow{\text{reduce}} \frac{244}{50} \to \frac{122}{25}$$

5.2 the Definite Integral

$$\int_a^b f(x)\,dx = \overset{(\#)}{(\text{signed})} \text{ area under the graph (curve) of } f(x)$$



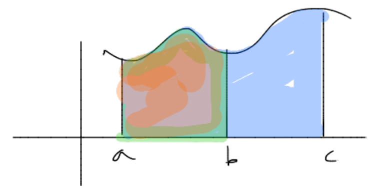
Blue region counts as $\oplus$

Green region count as $\ominus$

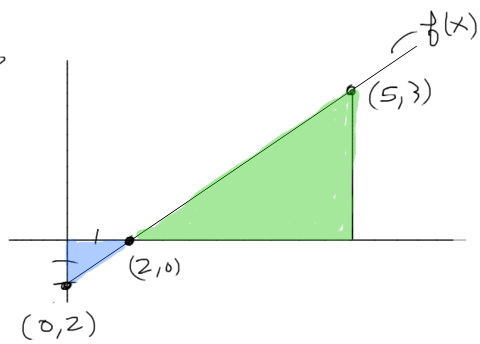
Key properties:

(Break it up)

$$\underbrace{\int_a^c f(x)\,dx}_{\text{entire shaded region}} = \underbrace{\int_a^b f(x)\,dx}_{\text{pinkish}} + \underbrace{\int_b^c f(x)\,dx}_{\text{blue}}$$



Ex,



$$\int_0^2 f(x)\,dx = -\left(\begin{matrix}\text{blue}\\ \text{area}\end{matrix}\right) = -\tfrac{1}{2}(2)(2) = -2$$

$$\int_2^5 f(x)\,dx = \tfrac{1}{2}(\text{base})(\text{height}) = \tfrac{1}{2}(3)\cdot 3 = \tfrac{9}{2} = 4.5$$

$(5-2)(3)$

How to compute $\int_a^b f(x)\,dx$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Fundamental theorem of Calculus

where F is an anti-derivative of $f(x)$

i.e., $\frac{d}{dx}\left(F(x)\right) = f(x)$

Examples: since $\frac{d}{dx}(x) = 1$ the anti-derivative of 1 is x.

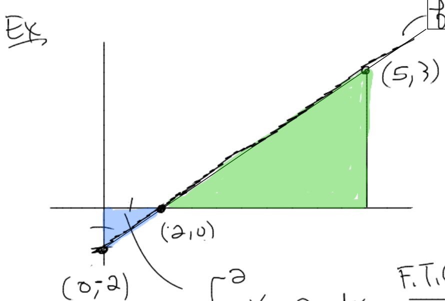
the anti-derivative of $3x$ is $\frac{3}{2}x^2$

$x^2 \xrightarrow{d/dx} 2x$

$3x^2 \to 6x$

$\frac{3}{2}x^2 \to \frac{3}{2}2x = 3x$

Ex.



① get eqn $f(x)$: $m = \frac{3-0}{5-2} = \frac{3}{3} = 1$ } $y - 0 = 1(x-2)$

point: (2,0) $\quad y = x-2$

$\underbrace{\tfrac{1}{2}(4) = 2}$ —— -2 —— $\underbrace{-4}$

negative ⇒ downstairs

"blue 2 = area"

$$\int_0^2 x - 2\,dx \overset{F.T.C}{=} \frac{1}{2}x^2 - 2x\Big|_0^2 = \left(\frac{1}{2}(2)^2 - 2(2)\right) - \left(\frac{1}{2}(0)^2 - 2(0)\right) = \boxed{-2}$$

↑ fcn whose $\frac{d}{dx} = x$

$\frac{d}{dx}\left(\frac{1}{2}x^2\right) = \frac{1}{2}\cdot 2x = x$

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② $\int_{\min x}^{\max x} \text{function}\,dx = \int_2^5 x - 2\,dx = \frac{1}{2}x^2 - 2x\Big|_2^5 =$ eval @ $x=5$ —— eval @ $x=2$

—green—

$= \left[\frac{1}{2}(5)^2 - 2(5)\right] - \left[\frac{1}{2}(2)^2 - 2(2)\right]$

$= \frac{1}{2}\cdot(25) - 10 \quad - \left[\underbrace{2-4}_{-2}\right] \quad 4.5$

$= 12.5 - 10 + 2 \quad = 14.5 - 10$

same # we found when computing area using $\frac{1}{2}(b)(h)$