Mon Wk 10

5.1.2

$f(x) = 3x^2 + 8x$ over $[-1, 1]$. ($a = -1$, $b = 1$)

R_5: $n = 5$
right endpoints

① $\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{5} = \frac{2}{5}$

②

-1 $-1+\frac{2}{5} = -\frac{3}{5}$ $-1+\frac{2}{5}(2) = -\frac{1}{5}$ $-1+\frac{2}{5}(3) = \frac{1}{5}$ $-1+\frac{2}{5}(4) = \frac{3}{5}$ $-1+\frac{2}{5}(5) = 1$

$x_1 = -\frac{3}{5}$, $x_2 = -\frac{1}{5}$, $x_3 = \frac{1}{5}$, $x_4 = \frac{3}{5}$, $x_5 = 1$

Since we want R_5 choose these (skip first)

L_5 (skip last)

M_5 (midpoints, eg $\frac{-1 + \left(-\frac{3}{5}\right)}{2} = \frac{-\frac{8}{5}}{2} = -\frac{8}{5} \cdot \frac{1}{2} = -\frac{4}{5}$) 1st midpt.

③ $R_5 = \sum_{i=1}^{5} \underbrace{\overset{\text{const}}{\Delta x} \cdot f(x_i)}_{\text{area of rectangle}} = \Delta x \left[f\left(-\frac{3}{5}\right) + f\left(-\frac{1}{5}\right) + f\left(\frac{1}{5}\right) + f\left(\frac{3}{5}\right) + f(1) \right]$

$f(x) = 3x^2 + 8x$

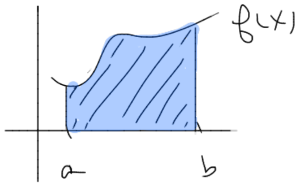
④ $\frac{2}{5}\left[3\left(-\frac{3}{5}\right)^2 + 8\left(-\frac{3}{5}\right) + 3\left(-\frac{1}{5}\right)^2 + 8\left(-\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + 8\left(\frac{1}{5}\right) + 3\left(\frac{3}{5}\right)^2 + 8\left(\frac{3}{5}\right) + 3(1)^2 + 8(1)\right]$

⑤ note: entering exact answer: 4.285

$4.285 = 4 + \frac{285}{1000} = \frac{4000 + 285}{1000} = \frac{4285}{1000} \xrightarrow{\text{reduce}}$

The Definite Integral

- the precise* area under the curve.

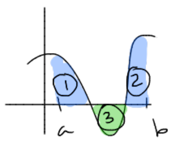


$$\int_a^b f(x)\,dx \overset{\text{def'n}}{=\!=} \lim_{N\to\infty} \overbrace{\sum_{i=1}^{N} \Delta x \cdot f(x_i)}^{\text{approx. area}}$$

$$= \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i) \cdot \underbrace{\Delta x}_{\text{small change in } x}$$

dx = differential
= infinitesimal change in x

* = signed. (area below x-axis is counted as negative)

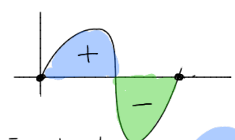


Regions ①, ② give + definite integrals

③ give −

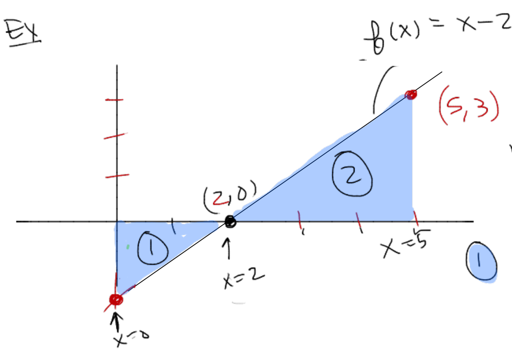
Ex

$$\int_0^{2\pi} \sin x\,dx \overset{\text{Fundamental theorem of Calculus}}{=} -\cos(x)\Big|_0^{2\pi} = -\overbrace{\cos(2\pi)}^{=1} - \Big(-\overbrace{\cos(0)}^{1}\Big) = -1-(-1) = 0$$



(what function has it's derivative = $\sin(x)$?

Ex



Calculate area shaded:

② $\int_2^5 x-2\,dx =$

$m = \frac{3-0}{5-2} = \frac{3}{3} = 1$

$y - 0 = 1(x-2) \Rightarrow y = x-2$

what fcn has deriv. = x − 2?

① $= \int_0^2 f(x)\,dx = \int_0^2 x-2\,dx = \frac{x^2}{2} - 2x\Big|_0^2 = \frac{2^2}{2} - 2(2) - \left[\frac{0^2}{2} - 2(0)\right]$

F.T.C.

$= \frac{4}{2} - 4 - 0 = -2$

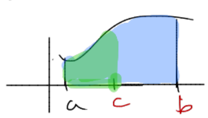
$x^2 \longrightarrow 2x$

$\frac{x^2}{2} \longrightarrow \frac{2x}{2} = x$

$-2x \longrightarrow -2$

Use: key property of $\int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx = \overbrace{\int_a^c f(x)\,dx}^{\text{green}} + \overbrace{\int_c^b f(x)\,dx}^{\text{blue}}$$



Entire shaded = green + blue

② $\frac{1}{2}(3)(3)$

$= \frac{9}{2} = 4.5$

② $\int_2^5 x-2\,dx = \frac{x^2}{2} - 2x\Big|_2^5 = \frac{5^2}{2} - 2\cdot5 - \left[\frac{2^2}{2} - 2\cdot2\right]$

$= \frac{25}{2} - 10 - (\underbrace{2-4}_{-2})$

$= 12.5 - 10 + 2 = 14.5 - 10 = 4.5$