Tue - wk 10

Today (5.3) the Indefinite Integral (general anti-derivative)

$\int f(x)\,dx$ = a function* *(infinite family of functions)

= all possible functions whose derivative is $f(x)$

Ex $\int 2\,dx$ = all functions whose derivative is = 2.

$= 2x$

or

$= 2x + 15^2$

in general

$= 2x + C$ w/ C = constant

$\frac{d}{dx}(x) = 1$ ②

$\frac{d}{dx}(2x) = 2$

Ex $\int \cos(x)\,dx = \sin(x) + C$

Ex $\int \sin(x)\,dx = -\cos(x) + C$

Ex $\int \sec(x)\tan(x)\,dx = \sec(x) + C$

Ex $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln|x| + C$ domain of ln is positive

Ex $\int x\,dx = \frac{1}{2}x^2 + C$

Ex $\int x^2\,dx = \frac{1}{3}x^3 + C$

Ex $\int x^3\,dx = \frac{x^4}{4} + C$

Power Rule for Integration

$\frac{1}{2} \cdot d/dx\,(x^2) = \frac{1}{2} \cdot 2x$

$\frac{d}{dx}\left(\frac{1}{2}x^2\right) = x$

$\frac{1}{3}\frac{d}{dx}(x^3) = \frac{1}{3}\,3x^2$

$\frac{d}{dx}\left(\frac{1}{3}x^3\right) = x^2$

Basic Integrals Chart

Functions	Their Anti-derivative
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$\sec^2(x)$	$\tan(x)$
e^x	e^x
$\csc^2(x)$	$\cot(x)$

Functions	Their Anti-derivative
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$
$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\sec^{-1}(x)$

Ex

$$\int \underbrace{\sqrt{x^3} + x^{-5/7}}_{\text{integrand}} \, dx = \int \sqrt{x^3}\, dx + \int x^{-5/7} dx$$

use "break it up property": $\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

Note! Not true for mult, & div.

$$= \int (x^3)^{\frac{1}{2}} dx + \int x^{-5/7} dx \underset{\text{power rule}}{=} \frac{x^{\frac{3}{2}+\frac{2}{2}}}{\frac{3}{2}+\frac{2}{2}} + \frac{x^{-\frac{5}{7}+\frac{7}{7}}}{\frac{2}{7}} = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{2}{7}}}{\frac{2}{7}} = \boxed{\frac{2}{5}x^{\frac{5}{2}} + \frac{7}{2}x^{\frac{2}{7}} + C}$$

$(x^3)^{\frac{1}{2}} = x^{3/2}$

check: $\frac{d}{dx}(\text{ans}) = \underbrace{\frac{2}{5} \cdot \frac{5}{2}}_{=1} x^{\frac{5}{2}-\frac{2}{2}} + \underbrace{\frac{7}{2} \cdot \frac{2}{7}}_{=1} x^{\frac{2}{7}-\frac{7}{7}} + 0 = \underline{x^{\frac{3}{2}} + x^{-\frac{5}{7}}}$

☺

Ex $\int 1\, dx = x + C$

Integrals

Indefinite Integral = function (family)

$\int f(x)\,dx$

Definite Integral = a number

= area under the curve

$\int_a^b f(x)\,dx = F(b) - F(a)$ w/ F = some anti-derivative of $f(x)$

find an anti-derivative, sub in b, sub in a, subtract

$\frac{9^2 \cdot 9}{3} = 9^2 \cdot 3 = 81 \cdot 3 = 243$

Ex $\int_1^9 x^2 + \sqrt{x}\,dx = \frac{x^3}{3} + \frac{2}{3}x^{3/2} \Big|_{x=1}^{x=9} = \left(\frac{9^3}{3} + \frac{2}{3}(9)^{3/2}\right) - \left(\frac{1^3}{3} + \frac{2}{3}(1)^{3/2}\right)$

$x=9$ $x=1$

$\sqrt{x} = x^{1/2} \rightarrow \frac{x^{1/2+2/2}}{\frac{3}{2}} = \frac{2}{3} \cdot x^{3/2}$

$= (243 + 18) - \left(\frac{1}{3} + \frac{2}{3}\right) = 261 - 1 = 260$

why not keep C in $\int_a^b f(x)\,dx$

Ex

$$\int_1^4 x^2\,dx = \frac{x^3}{3} + C \Big|_1^4 = \frac{4^3}{3} + C - \left(\frac{1^3}{3} + C\right) = \frac{4^3}{3} - \frac{1}{3}$$

C's cancel

$$\int_7^8 \frac{dx}{x} = \int_7^8 \frac{1}{x}\,dx$$

$$I = \int_0^9 |x^2 - 1|\,dx = \int_0^1 |x^2-1|\,dx + \int_1^9 |x^2-1|\,dx$$

$$\int_0^1 -(x^2-1)\,dx + \int_1^9 x^2-1\,dx$$

$$|x^2-1| = \begin{cases} x^2-1 & \text{when } x^2-1>0 \\ -(x^2-1) & \text{when } x^2-1<0 \quad (\quad x \in (-1,1)) \end{cases}$$

(solve this

$x^2-1=0$

$x = \pm 1$