Thu - Wk 10

5-3 Indefinite Integral or general anti-derivative

$$\int f(x)\,dx = \text{all possible functions whose derivative is } f(x).$$

Ex.

$$\int 2\,dx = \text{all functions whose derivative is } = 2.$$

$$= 2x + 21^{2}$$

$$= 2x + C \quad (\text{where } C = \text{constant})$$

$2x \to 2$

Ex $\int 5\sec(x)\tan(x)\,dx = 5\sec(x) + c$

$\frac{d}{dx}(5\sec(x)) = 5\sec(x)\tan(x)$ ✓

Ex $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$

Ex $\int \overbrace{7\cdot\cos(x)}^{\text{integrand}}\,dx = 7\int \cos(x)\,dx = 7\cdot\sin(x) + C$

$\boxed{\text{Note: } \int 7 + \cos(x)\,dx \neq 7 + \int \cos(x)\,dx}$

$\frac{d}{dx}(7\sin x) = 7\cos x$ ☺

Ex $\int 3x + 1\,dx = \int 3x\,dx + \int 1\,dx$

Key Property: "break it up"

$\frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{2}\cdot 2x = x$

$= 3\int x\,dx + \int 1\,dx$

$= 3\cdot\frac{x^2}{2} + C_1 + x + C_2$

$= \frac{3x^2}{2} + x + C$ accumulate all consts

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Note: not for $*$ or $\div$ (recall product quotient rule)

Power Rule

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n = \text{constant}, \; n \neq -1)$$

kick the exponent up by one , then divide by it.

Ex

$$\int x^1 \, dx = \frac{x^2}{2} + c$$

$$\int x^3 \, dx = \frac{x^4}{4} \times c$$

$$\frac{1}{4} \cdot \frac{d}{dx}(x^4) = \frac{4x^3}{4} \Rightarrow \frac{d}{dx}\left(\frac{x^4}{4}\right) = \frac{4x^3}{4} = x^3$$

$n = -1$ case

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

domain of $\ln(x)$ is $x > 0$ positive

Basic Integrals Chart

Function	Anti-derivative
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\|x\|$
e^x	e^x
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\sec(x)\tan(x)$	$\sec(x)$
$\sec^2(x)$	$\tan(x)$

Function	Anti-derivative
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$
$\frac{1}{\|x\|\sqrt{x^2-1}}$	$\sec^{-1}(x)$

Ex

$$\int \sqrt{x^3} + x^{-5/7}\,dx$$

$$\downarrow$$

$$\int (x^3)^{\frac{1}{2}} + x^{-5/7}\,dx$$

$$\downarrow$$

$$\int x^{\frac{3}{2}} + x^{-\frac{5}{7}}\,dx = \frac{x^{\frac{3}{2}+\frac{2}{2}}}{\frac{3}{2}+\frac{2}{2}} + \frac{x^{-\frac{5}{7}+\frac{7}{7}}}{\frac{2}{7}} = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{2}{7}}}{\frac{2}{7}} = \frac{2}{5}x^{\frac{5}{2}} + \frac{7}{2}x^{\frac{2}{7}} + C$$

both powers,
(+) $\Rightarrow$ piece by piece

check: $$\frac{d}{dx}\left(\frac{2}{5}x^{\frac{5}{2}} + \frac{7}{2}x^{\frac{2}{7}} + C\right) = \underbrace{\frac{2}{5}\cdot\frac{5}{2}}_{=1}\cdot x^{\frac{5}{2}-\frac{2}{2}} + \underbrace{\frac{7}{2}\,\frac{2}{7}}_{=1}x^{\frac{2}{7}-\frac{7}{7}} = \underbrace{x^{\frac{3}{2}} - x^{-\frac{5}{7}}}_{\text{integrand}}$$

power

☺

Fundamental Theorem of Calculus (Part I)

$$\int f(x)\,dx = \text{a function (infinite family of .... + C)}$$

$$\int_a^b f(x)\,dx = \text{a number} = \text{area under graph of } y = f(x)$$

$$= F(b) - F(a) \quad \text{w/ } F = \text{some anti-derivative of } f(x)$$

integrate f(x), then plug in b, plug in a, subtract

Ex $$\int_{-3}^{5} x^3 + 5x\,dx = \frac{x^4}{4} + \frac{5x^2}{2}\Big|_{-3}^{5} = \frac{5^4}{4} + \frac{5(5)^2}{2} - \left(\frac{(-3)^4}{4} + \frac{5(-3)^2}{2}\right) = 176$$