Name: ______________________________ **Exam 3 Study Guide** March 23, 2024

Find the indicated antiderivative. CHECK YOUR ANSWERS

1. $\int \frac{3x^2}{\sqrt{x^3-1}}\,dx = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = 2u^{1/2} + c$

$= \boxed{2\sqrt{x^3-1} + c}$

$u = x^3 - 1$

$du = 3x^2\,dx$

2. $\int \frac{e^{\cot(x)}\csc(x)}{\sin(x)}\,dx = \int e^u\,du$

check:

$\frac{d}{dx}\left(-e^{\cot x}\right) = -e^{\cot(x)} \cdot (-\csc^2 x)$

$= e^{\cot(x)} \cdot \csc x \cdot \csc x$

$= e^{\cot x} \cdot \csc x \cdot \frac{1}{\sin x}$

$\boxed{u = \cot x}$ $\xleftrightarrow{\text{related}}$ $\tan x$ $\xrightarrow{d/dx}$ $\sec^2 x$

$\frac{du}{dx} = -\csc^2 x$

$= -\int e^u\,du = \boxed{-e^{\cot(x)} + c}$

$du = -\csc^2 x\,dx$

sub $\int \frac{e^u \csc(x)}{\sin(x)} \frac{-1}{\csc^2(x)}\,du = \int \frac{e^u\,(-1)}{\sin(x)\cdot\csc(x)}\,du$

$\boxed{\frac{1}{-\csc^2 x}\,du = dx}$

3. $\int 6x^2 \sin(x^3)\cos(x^3)\,dx = \int 6x^2 \sin(u)\cos(u)\frac{1}{3x^2}\,du = 2\int \sin(u)\cos(u)\,du$

$\boxed{u = x^3}$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\boxed{\frac{1}{3x^2}\,du = dx}$

$w = \sin(u)$

$dw = \cos(u)\,du$

$= 2\int w\,dw = \frac{2w^2}{2} + c$

$= (\sin(u))^2 + c$

$= (\sin(x^3))^2 + c$

$w = \cos(u)$

$dw = -\sin(u)\,du$

$\frac{1}{-\sin(u)}\,dw = du$

$2\int \sin(u) \cdot w \cdot \frac{1}{-\sin(u)}\,dw$

$-2\int w\,dw = -2\frac{w^2}{2} + c$

$\frac{d}{dx}(\text{ans}) =$

$-2(\cos(x^3))' \cdot (-\sin(x^3) \cdot 3x^2$

$= 6x^2\cos(x^3)\sin(x^3)$

$-(\cos(u))^2 + c$

$= \boxed{-(\cos(x^3))^2 + c}$

check $\frac{d}{dx}(\text{ans}) = 2(\sin(x^3))' \cdot \cos(x^3) \cdot 3x^2$

$$\cot x \xrightarrow{d/dx} -\csc^2 x$$

$\parallel$ set u

5. $\int \cot^3 x \csc^2 x \, dx = \int u^3 \csc^2 x \left(\frac{1}{-\csc^2 x} \right) du = -\int u^3 du$

$u = \cot x$

$$= -\frac{u^4}{4} + c$$

$$\frac{du}{dx} = -\csc^2 x$$

$$= \boxed{-\frac{\cot^4 x}{4} + c}$$

$$du = -\csc^2 x \, dx$$

$$-\sin^2 x \, du = \frac{1}{-\csc^2 x} du = dx$$

7. $\int \frac{x}{x+3}\,dx =$

NO deg 1 diffs
NO deriv. rel.
NO common trig deriv.
when in doubt set $u =$ most complicated sub-expression

$u = x+3$

$du = dx$

$\underset{\text{sub}}{=} \int \frac{x}{u}\,du = \int \frac{u-3}{u}\,du = \int \frac{u}{u}\,du - \int \frac{3}{u}\,du$

stuck b/c variables mixed,
mine $u = x+3$
$\Rightarrow \boxed{u - 3 = x}$

$= \int 1 \cdot du - \int \frac{3}{u}\,du$

$= u - 3\ln|u| + C$

$= \boxed{x+3 - 3\ln|x+3| + C}$

check!

$\frac{d}{dx}(\text{ans}) = 1 - 3\left(\frac{1}{x+3}\right)\cdot 1 = 1 - \frac{3}{x+3} = \frac{x+3}{x+3} - \frac{3}{x+3} = \frac{x}{x+3}$

4. $\displaystyle\int \frac{2x+1}{x^2+1}\,dx = \int \frac{2x+1}{u} \cdot \frac{1}{2x}\,du = \int \frac{1+\frac{1}{2x}}{u}\,du$

mixed var. $\Rightarrow$:(

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$\displaystyle\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$

common trig deriv.

$\displaystyle\int \frac{1}{u}\,du +$

$\int u^5\,du$

3. $\displaystyle\int \tan^5 x \overbrace{\sec^2 x\, dx}$ = $\dfrac{u^6}{6} + c$ = $\boxed{\dfrac{\tan^6(x)}{6} + c}$

$u = \tan x$

$du = \sec^2 x\, dx$

check

$\dfrac{d}{dx}(\text{ans})$ $\dfrac{6}{6}(\tan x)^5 \cdot \sec^2 x$ = :)

$\sin^2 x + \cos^2 x = 1 \Longrightarrow \sin^2 x = 1 - \cos^2 x$

$$\int \underbrace{\left(1 - \cos^2(x)\right)}\cos(x)\,dx = \int \cos(x) - \cos^3(x)\,dx$$

$$= \int \sin^2(x)\cdot\cos(x)\,dx$$

$u = \sin x$

$du = \cos x\, dx$

$$= \int u^2\,du = \boxed{\frac{\sin^3 x}{3} + c}$$