4.6.2

$$f(x) = \frac{x-2}{x-7}$$

vertical asy: ↕ $x = c$ eg, $x = 7$

horizontal asy: $y = c$

$$\lim_{x \to \infty} f(x) = \frac{x-2}{x-7} \overset{\text{L'H}}{=} 1$$

$y = 1$

Definite integral warm-up

$$\int_{-1}^{5} 4x^2 + 6\,dx = \frac{4x^3}{3} + 6x \Big|_{-1}^{5} = \left(\quad\right) - \left(\quad\right)$$

@ $x = 5$ @ $x = -1$

$$= \left(\frac{4(5)^3}{3} + 6\cdot 5\right) - \left(\frac{4(-1)^3}{3} + 6(-1)\right)$$

4:12:25

$$= \frac{500}{3} + 30 - \left(-\frac{4}{3} - 6\right) = \frac{500}{3} + 30 + \frac{4}{3} + 6$$

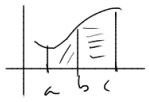
$$= \frac{504}{3} + 36 = 204$$

$168 + 36$

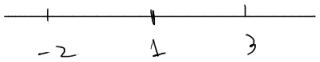
Evaluate the integral $\int_{-2}^{3} f(x)\,dx$, where

$$f(x) = \begin{cases} 1 - x^3, & x \le 1 \\ x^2, & x > 1 \end{cases}$$

key: $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$



(1)



$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx = \int_{-2}^{1} 1 - x^3\,dx + \int_{1}^{3} x^2\,dx$$

$[-2, 1] \Rightarrow x \le 1$ $\quad [1, 3]$ $x > 1$

$$= x - \frac{x^4}{4}\Big|_{-2}^{1} + \frac{x^3}{3}\Big|_{1}^{3}$$

$-2 \wedge 4 = -16$

$(-2) \wedge 4$

$$= \left(1 - \frac{1}{4}\right) - \left(-2 - \frac{(-2)^4}{4}\right) + \frac{3^3}{3} - \frac{1^3}{3}$$

@ $x = 1$ @ $x = -2$

$$= \frac{3}{4} - (-6) + 9 - \frac{1}{3} = 15.75 - .\overline{3} = 15.41\overline{7}$$

(5.7) u- substitution

Consider

$$\int (3x+1)^2\,dx \neq \frac{(3x+1)^3}{3} + C$$

1. where on the chart does this match?
yes, x^n, but not EXACTLY

$$\int 9x^2 + 6x + 1\,dx$$

$$9\int x^2\,dx + 6\int x^1\,dx + \int 1x^0\,dx$$

$$\frac{9x^3}{3} + \frac{6x^2}{2} + x + C$$

Key: nothing special about "x"

need: $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

transform given problem into this

① set $u = 3x+1$

② $\frac{d}{dx}(u) = \frac{d}{dx}(3x+1)$

③ $du = 3\,dx$

④ $\frac{1}{3}du = dx$

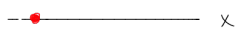
$\frac{du}{dx} = 3$ — ratio of differentials — infinitesimal change in variable

Basic Integrals Chart

Function	Anti-derivative	Function
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\frac{1}{1+x^2}$
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\frac{1}{\sqrt{1-x^2}}$
e^x	e^x	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\sin(x)$	$-\cos(x)$	
$\cos(x)$	$\sin(x)$	
$\sec(x)\tan(x)$	$\sec(x)$	
$\sec^2(x)$	$\tan(x)$	

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

here this x is some real #



$$\int (u)^2 \frac{1}{3}\,du = \frac{1}{3}\int u^2\,du = \frac{1}{3}\cdot\left(\frac{u^3}{3} + C\right)$$

mult. constant pull out

$$\frac{(3x+1)^3}{9} + C$$

$$= \frac{1}{3}\cdot\frac{(3x+1)^3}{3} + C$$

Ex

$\int (5x+1)^3\,dx$

① chart? NO ② similar to u^n, set $u = 5x+1$

③ $\frac{du}{dx} = 5$ ④ $du = 5\,dx$ ⑤ $\frac{1}{5}\,du = dx$

$\int u^3 \cdot \frac{1}{5}\,du = \frac{1}{5}\int u^3\,du = \frac{1}{5}\cdot\frac{u^4}{4} + C$ ← make-up step

← set x back $\frac{1}{20}(5x+1)^4 + C$

check

$\frac{d}{dx}(\text{ans}) = \frac{4}{20}(5x+1)^3 \cdot 5 = (5x+1)^3$?

$\int \cos(7x)\,dx$

$\int (5x-1)^{-1}\,dx$

$\int x(3x^2+1)^4\,dx$ think of which part of chart?

$u = 3x^2+1$ $\quad \frac{du}{dx} = 6x$ $\quad du = 6x\,dx$ $\quad \frac{1}{6x}\,du = dx$

$\int x \cdot (u)^4 \frac{1}{6x}\,du = \frac{1}{6}\int u^4\,du = \frac{1}{6}\frac{(3x^2+1)^5}{5} = \frac{(3x^2+1)^5}{30} + C$