Thu wk 10

$$\int_{-2}^{5} x^{1/3}\,dx = \left.\frac{x^{4/3}}{4/3}\right|_{-2}^{5}$$

$$= \left.\frac{3}{4}\cdot x^{4/3}\right|_{-2}^{5} = \frac{3}{4}\left[5^{4/3} - (-2)^{4/3}\right]$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\underbrace{-2\wedge(4/3)}_{-\left(2^{4/3}\right)}$$

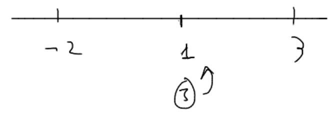
$(-2)\wedge(4/3)$

Evaluate the integral $\int_{-2}^{3} f(x)\,dx$, where

$$f(x) = \begin{cases} 1 - x^3, & x \le 1 \\ x^2, & x > 1 \end{cases}$$

↑ 2

1



power rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx = \int_{-2}^{1} 1 - x^3\,dx + \int_{1}^{3} x^2\,dx$$

where do these x-vals live?

$x \le 1$

>1

$$= \left(x - \frac{x^4}{4}\right)\Big|_{-2}^{1} + \frac{x^3}{3}\Big|_{1}^{3} = \left(1 - \frac{1^4}{4}\right) - \left(-2 - \frac{(-2)^4}{4}\right) + \left(\frac{3^3}{3} - \frac{1^3}{3}\right) =$$

$$= \frac{3}{4} - (\underbrace{-2-4}_{-6}) + \left(9 - \frac{1}{3}\right) = \frac{3}{4} + 6 + 9 - \frac{1}{3} = 15.75 - .\overline{3}$$

$$= 15.41\overline{7}$$

Question 7 of 10

Calculate $F(8)$ given that $F(5) = 2$ and $F'(x) = x^2$. *Hint:* Express $F(8) - F(5)$ as a definite integral.

(Use symbolic notation and fractions where needed.)

$$\boxed{\underbrace{F(8)}_{\text{goal: (isolate)}} - \overbrace{F(5)}^{=2} = \int_5^8 f(x)^{=x^2}\,dx}$$

F. T. C.

$$(w)\ F' \overset{=x^2}{} = f\ )$$

$$F(b) - F(a) = \int_a^b f(x)\,dx$$

$$1024$$

$$8^3 = (2^3)^3 = 2^9 = \frac{512}{3} - \frac{125}{3}$$

$$F(8) - F(5) = F(8) - 2 = \int_5^8 x^2\,dx = \frac{x^3}{3}\Big|_5^8 = \frac{8^3}{3} - \frac{5^3}{3} = \frac{387}{3}$$

$$131$$

$$F(8) = \frac{387}{3} + 2 \cdot \frac{3}{3} = \frac{393}{3} = 131$$

Indefinite Integrals

$$\int 5x + 4 \, dx = 5\frac{x^2}{2} + 4x + C$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (x^3+1)^2 \, dx = \frac{(x^3+1)^3}{3}$$

There's nothing special about x

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$$

Basic Integrals Chart

Function	Anti-derivative	Function	
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\frac{1}{1+x^2}$	
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\frac{1}{\sqrt{1-x^2}}$	
e^x	e^x		
$\sin(x)$	$-\cos(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	
$\cos(x)$	$\sin(x)$		
$\sec(x)\tan(x)$	$\sec(x)$		
$\sec^2(x)$	$\tan(x)$		

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int (x^3+1)^2 \, dx \underset{\text{expand}}{\overset{1}{=}} \int x^6 + 2x^3 + 1 \, dx$$

$$= \frac{x^7}{7} + \frac{2x^4}{4} + x + c$$

chain rule $\quad u^2 \rightarrow 2u \cdot du$

$$\frac{d}{dx}\left((x^3+1)^2\right) = 2(x^3+1) \cdot 3x^2$$

$$\int (3x+1)^2\,dx \neq \frac{(3x+1)^3}{3} + c$$

why?

$$\frac{d}{dx}(\text{ans}) = \frac{1}{3}\cdot\cancel{3}(3x+1)^2\cdot 3$$

$$= (3x+1)^2\cdot 3$$

not the integrand

Fix:

u-substition

① set $\boxed{u = (3x+1)}$

② $\frac{d}{dx}(u) = \frac{d}{dx}(3x+1) = 3$

$\quad\ \ \| $

ratio of differentials $\frac{du}{dx}$

③ isolate dx

$$dx\cdot\frac{du}{dx} = 3\cdot dx \implies du = 3\cdot dx$$

$$\boxed{\frac{1}{3}du = dx}$$

$$\int u^2\,\frac{1}{3}\,du \;\underset{\text{pull-out mult. const.}}{=\!=\!=}\; \frac{1}{3}\int u^2\,du$$

chart $(u \leftrightarrow x)$

$$= \frac{1}{3}\cdot\frac{u^3}{3} + C$$

$$= u^3/9 + C = \frac{(3x+1)^3}{9} + C$$

Ex.

does the integrand match chart ? … no
what pieces of the integrand match chart? … cos
think this one is

$$\int \cos(3x)\,dx \qquad \int \cos(u)\,du = \sin(u) + C$$

Q: transform

when in doubt, set u = what's inside parenthesis

$$\boxed{u = 3x} \qquad du = 3\,dx$$

$$\frac{du}{dx} = 3 \qquad \boxed{\tfrac{1}{3}du = dx}$$

$$\int \cos(u)\tfrac{1}{3}\,du$$

$$= \tfrac{1}{3}\int \cos(u)\,du \underset{\text{chart}}{=} \tfrac{1}{3}\cdot\sin(u) + C \underset{\text{get x-back}}{=} \tfrac{1}{3}\sin(3x) + C$$

check:

$$\frac{d}{dx}\left(\tfrac{1}{3}\sin(3x)\right)$$
$$= \tfrac{1}{3}\cos(3x)\cdot 3$$
$$= \cos(3x)$$

$$\int (5x+1)^{-1}\,dx$$
$$=$$