Wed. Wk 10

HW

4.3.5

$f(x) = 4xe^{-x^2}$

$f'(x) = 4e^{-x^2} + 4x \cdot e^{-x^2}(-2x) = 4e^{-x^2}\left[1-2x^2\right]$

$f''(x) = 4e^{-x^2} \cdot (-2x)\left[1-2x^2\right] + 4e^{-x^2}\left[\overset{-2x\cdot(2)}{-4x}\right]$

$= \underbrace{4e^{-x^2}(-2x)}_{\text{I}}\left[\underbrace{(1-2x^2}_{\text{II}} + 2)\right] \overset{\text{P.O.I}}{=} 0$

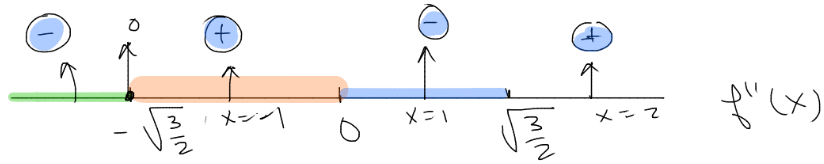
$\text{I} = 0 = 4e^{-x^2}(-2x) = -8xe^{-x^2}$ zero property again

$-8x = 0 \Rightarrow x = 0$

or $e^{-x^2} = 0 \Rightarrow -x^2 = \ln 0$ DNE (No sols)

$\text{II} = 0 = \underbrace{3-2x^2}$

$2x^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{2}}$



P.O.I

$x = 0$

$x = \pm\sqrt{\frac{3}{2}}$

Asymptote (infinity behavior)

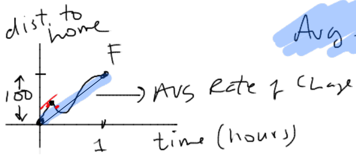
Horizontal (what are the y-values when $x \longrightarrow \infty$

$\lim_{x\to\infty} 4xe^{-x^2} = \lim_{x\to\infty} \frac{4x}{e^{x^2}} \overset{\text{D.S.}}{=} \frac{\infty}{\infty}$ L'H

L'H $\lim_{x\to\infty} \frac{4}{e^{x^2} \cdot 2x} = \frac{4}{\infty} = 0$:)

The Fundamental Theorem of Calculus

Key Tool: Mean Value theorem.

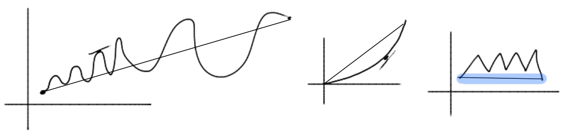


Avg. Rate of Change is achieved @ least once @ some instant

FOR CONTINUOUS FUNCTIONS !! & smooth (differentiable)

$$\boxed{\frac{F(b)-F(a)}{b-a} = F'(c)}$$

$$F(b)-F(a) = F'(c)(b-a)$$



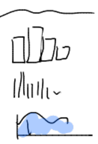
(Part I)

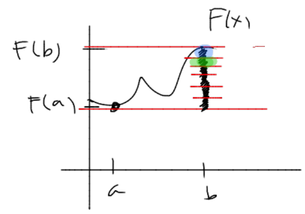
$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

$\boxed{\text{w/ } F = \text{some anti-derivative of } f(x).}$

area

$$\lim_{n\to\infty} \sum_{i=0}^{n} f(x_i)\Delta x$$





$$= F(b) - F(x_1) + F(x_1) - F(x_2) + F(x_2) - F(x_3) + \dots + F(a)$$

$b = x_0$, $a = x_k$

$$= \sum_{i=0}^{k} F(x_i) - F(x_{i+1}) \overset{M.V.T}{=} \sum_{i=0}^{k} F'(x_i)\cdot\Delta x$$

$\Delta x = x_i - x_{i+1}$

now take limit $k \to \infty$

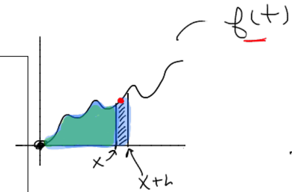
$$= \lim_{k\to\infty} \sum_{i=0}^{k} F'(x_i)\cdot\Delta x = \int_a^b F'(x)\,dx = \int_a^b f(x)\,dx$$

F.T.C. Part II

Area so far function

$$A(x) = \int_0^x f(t)\,dt$$

$$\frac{d}{dx}\left(A(x)\right) = A'(x) = \underline{\quad f(x) \quad}$$



time

$$A'(x) \approx \frac{A(x+h) - A(x)}{h} = \frac{\text{Area}(\;)}{h} = \frac{f(x)\cdot \cancel{h}}{\cancel{h}}$$

take limit $\Rightarrow$ $A'(x) = f(x)$