Name: ______________________________ **Exam 3** March 30, 2024

Find the indicated antiderivative. Show your work

1. Let $f''(x) = 3x + 2$ and assume $f'(2) = 11$ and $f(4) = 1$. Determine $f(x)$.

$f'(x) = \int f''(x)\,dx = \int 3x + 2\,dx = \frac{3x^2}{2} + 2x + C$, $f'(2) = \frac{3 \cdot 2^2}{2} + 2 \cdot 2 + C = 11$

$6 + 4 + C = 11 \Rightarrow C = 1$

$= \frac{3x^2}{2} + 2x + 1$

$\boxed{\frac{1}{2}x^3 + x^2 + x - 51 = f(x)}$

$f(x) = \int \frac{3x^2}{2} + 2x + 1\,dx = \frac{3}{3 \cdot 2}x^3 + \frac{2x^2}{2} + x + C$, $f(4) = 1$

$= \frac{1}{2}x^3 + x^2 + x + C$

$\frac{1}{2}(4)^3 + 4^2 + 4 + C = 1$

$32 + 16 + 4 + C = 1 \Rightarrow C = -51$

2. $\int \frac{3x + 5x^2 + 7}{x} dx =$

can't integrat factor by factor

algebra 1st : $\int \frac{3x}{x} + \frac{5x^2}{x} + \frac{7}{x}\,dx = \int 3 + 5x + \frac{7}{x}\,dx = \boxed{3x + \frac{5x^2}{2} + 7 \cdot \ln|x| + C}$

3. $\int x^3 \sqrt{x^5} dx =$

$\int x^3 \cdot x^{5/2}\,dx = \int x^{3 + 5/2}\,dx = \int x^{11/2}\,dx = \boxed{\frac{2}{13}x^{13/2} + C}$

4. $\int 3x \sin(x^2)\, dx = \int 3\cancel{x} \cdot \sin(u) \frac{1}{2\cancel{x}}\,du = \frac{3}{2}\int \sin(u)\,du = -\frac{3}{2}\cos(u) + C$

see 1 sin think $= \int \sin(u)\,du$

$\boxed{-\frac{3}{2}\cos(x^2) + C}$

do: $\boxed{u = x^2}$ | $du = 2x\,dx$

$\frac{du}{dx} = 2x$ | $\boxed{\frac{1}{2x}du = dx}$

5. $\int \frac{5x^4}{\sqrt{x^5 - 1}} dx = \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = \boxed{2(x^5 - 1)^{1/2} + C}$

see degree 1 diff

set $u =$ higher degree term, include constants

$u = x^5 - 1$

$du = 5x^4\,dx$

6. $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int \sin^{-1}(x) \cdot \frac{1}{\sqrt{1-x^2}}\,dx$

see deriv. relation

$u = \sin^{-1}(x)$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\sqrt{1-x^2}\,du = dx$

$= \int u \frac{1}{\sqrt{1-x^2}} \sqrt{1-x^2}\,du = \int u\,du = \frac{u^2}{2} + C$

$\boxed{\frac{(\sin^{-1}(x))^2}{2} + C}$

7. $\int \frac{\sec(x)}{e^{\tan(x)}\cos(x)}\,dx =$

or $\dashrightarrow \int \frac{e^{-\tan x} \cdot \sec(x)}{\cos(x)}\,dx$ $\quad u = -\tan x$

see 1 e: $u = \tan x$

$du = \sec^2 x\,dx$

$\frac{1}{\sec^2 x}\,du = dx$

$\int \frac{\sec x}{e^u \cos x} \cdot \frac{1}{\sec^2 x}\,du = \int \frac{1}{e^u}\,du = \int e^{-u}\,du$

$w = -u$

$dw = -du$

$-dw = du$

$\Rightarrow \int e^w(-dw) = -\int e^w\,dw = -e^w + C$

$= \boxed{-e^{(-\tan x)} + C}$

8. $\int \frac{2x-2}{x^2+1}\,dx =$

split up:

$\int \frac{2x}{x^2+1} - \frac{2}{x^2+1}\,dx = \int \frac{du}{u} - 2\underbrace{\int \frac{1}{x^2+1}\,dx}_{\tan^{-1}x}$

$u = x^2+1$

$du = 2x$

$\tan^{-1}x$

$\ln|u|$

$\boxed{\ln|x^2+1| - 2\tan^{-1}x + C}$

9. $\int x\sqrt{x-7}\,dx = \int x\sqrt{u}\,du = \int (u+7)\sqrt{u}\,du \overset{\text{algebra}}{=} \int u^{3/2} + 7u^{1/2}\,du$

same degree

mine

set $u = x-7$

$du = dx$

$x = u+7$

$= \frac{2}{5}u^{5/2} + \frac{7u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{5}(x-7)^{5/2} + \frac{14}{3}(x-7)^{3/2} + C}$

des. 1 diff

10. $\int 8x^3\sqrt{x^4-1}\,dx =$ $2\int 4x^3\sqrt{x^4-1}\,dx = 2\int \sqrt{u}\,du = 2\int u^{1/2}\,du$

$u = x^4 - 1$

$du = 4x^3\,dx$

$\frac{2 \cdot 2}{3} u^{3/2} + c = \boxed{\frac{4}{3}(x^4-1)^{3/2} + c}$

think

11. $\int \frac{3}{x\sqrt{x^8-1}}\,dx =$ $\int \frac{1}{|u|\sqrt{u^2-1}}\,du = \int \frac{3}{x\sqrt{u^2-1}} \cdot \frac{1}{4x^3}\,du$

no deg 1 diff

$\sec^{-1}$

$= \frac{3}{4}\int \frac{1}{x^4\sqrt{u^2-1}}\,du = \frac{3}{4}\int \frac{du}{|u|\sqrt{u^2-1}}$

set $(u^2 = x^8$

$u = x^4$

$du = 4x^3\,dx$

$\frac{1}{4x^3}du = dx$

$= \boxed{\frac{3}{4}\sec^{-1}(x^4) + C}$

12. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$ $\int \frac{e^u}{\sqrt{x}} \cdot 2x^{1/2}\,du = 2\int e^u\,du$

see 1 e

$u = \sqrt{x}$

$\boxed{2e^{\sqrt{x}} + c}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2x^{1/2}du = dx$

13. $\int \tan(x)\sec^2(x)\,dx =$ $\int u\,du = \frac{u^2}{2} + c = \boxed{\frac{\tan^2 x}{2} + c}$

$u = \tan x$

$du = \sec^2 x$

or

$\int \underset{u}{\sec(x)} \cdot \underbrace{\sec(x)\tan(x)}_{du} = \int u\,du = \frac{u^2}{2} + c$

$= \boxed{\frac{\sec^2(x)}{2} + c}$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

~~14.~~ $\frac{1}{2}\int \frac{2x}{\sqrt{1-x^4}}\,dx$ $\overset{\text{think}}{=} \int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}(x^2) + C$

no deriv. rel

$u^2 = x^4$

$u = x^2$, $du = 2x\,dx$

15. $\int \frac{\ln(2x)}{x}\,dx = \int \ln(2x)\cdot\frac{1}{x}\,dx = \int u\,du = \frac{u^2}{2} + c$ $\qquad \ln 2x = \underbrace{\ln 2}_{\text{const}} + \ln x$

$\frac{d}{dx}(\ln 2x) = \frac{1}{x}$ $\qquad \frac{2}{2x}$ $\qquad = \frac{\ln(2x)}{2} + C$

16. $-\int -\sin(x)\sqrt{1+\cos(x)}\,dx =$ $-\int u^{1/2}\,du = -\frac{3}{2}(1+\cos x)^{3/2} + C$

$u = 1 + \cos x$

$du = -\sin x$

17. $\int \frac{x}{\sqrt{x+4}}\,dx$ $\qquad \int \frac{x}{\sqrt{u}}\,du = \int \frac{u-4}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{4}{\sqrt{u}}\,du$

$u = x + 4$ $\qquad$ see # 2 $\qquad = \int u^{1/2} - 4u^{-1/2}$

$du = dx$

$\frac{2}{3}(x+4)^{\frac{3}{2}} - 8(x+4)^{\frac{1}{2}} + C$

18. Fill in the blank. One thing I think I'll remember from this class is

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