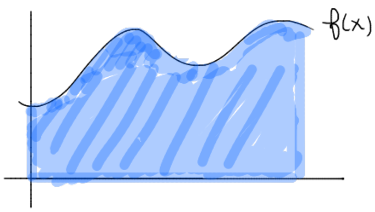
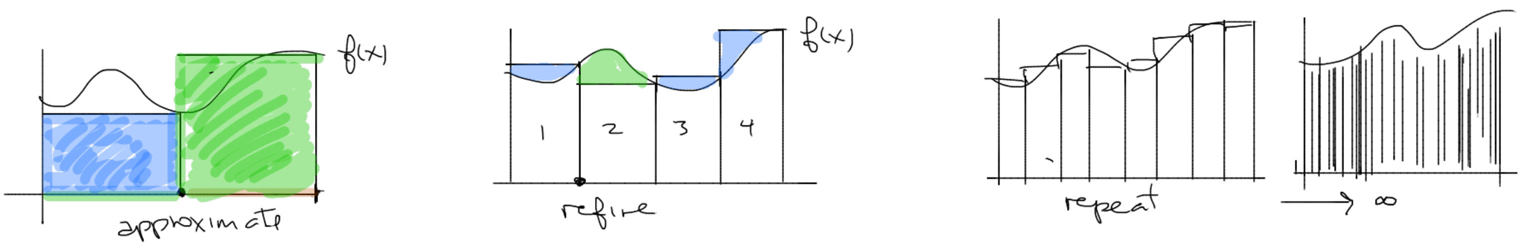
Applications

Motivating example: compute area under curve.



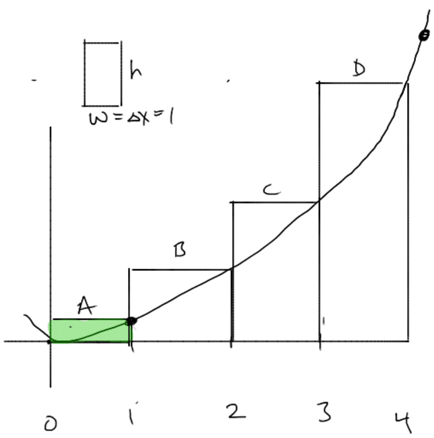
Idea: If $f(x)$ represents speed or velocity, the area under the curve represents distance traveled.

Reimann Sums



A detailed example

$f(x) = x^2$, region $= [0, 4]$, compute area under curve.



$(x, y) = (x, x^2)$ $n = 4$

use $\overbrace{4 \text{ rectangles}}^{n=4}$ to approximate area

$[0,4] = [a,b]$, $\Delta x = \dfrac{b-a}{n} = \dfrac{4-0}{4} = 1$ (width of rectangles)

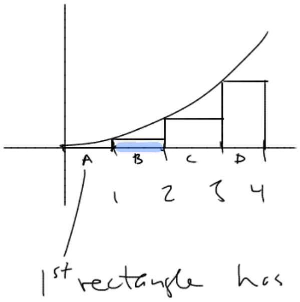
$$\text{Area} \approx A + B + C + D$$

$$A = f(1)\cdot\Delta x = 1\cdot 1,\quad B = f(2)\cdot\Delta x = 4\cdot 1,\quad C = f(3)\cdot\Delta x = 9\cdot 1,\quad D = f(4)\cdot\Delta x = 16\cdot 1$$

$$f(1)\cdot\Delta x + f(2)\cdot\Delta x + f(3)\cdot\Delta x + f(4)\cdot\Delta x = 1\cdot 1 + 4\cdot 1 + 9\cdot 1 + 16\cdot 1 = 30 \text{ sq. units.}$$

(Right Endpoint Method)

Left End Method



1^{st} rectangle has 0 ht

$$A + B + C + D = 0 + f(1)\cdot\Delta x + f(2)\Delta x + f(3)\Delta x$$

$$= 0 + 1 + 4 + 9 = 14$$

Our best guess: Averge two sol's

$\text{Avg}(30, 14) = 22$

To get precise area, increase # rectangles

Area#1 $\approx f(1)\Delta x + f(2)\Delta x + \dots + f(4)\Delta x = \sum_{i=1}^{4} f(i)\Delta x$ w/ $\Delta x = \frac{b-a}{n}$

Area#2 $\approx f(0)\Delta x + f(1)\Delta x + \dots + f(3)\Delta x = \sum_{i=0}^{3} f(i)\Delta x$

Increase # of rectangles by letting $n \to \infty$

$$\Delta x = \frac{b-a}{n} \longrightarrow dx$$

Fundamental theorem of Calculus

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \text{Area under curve above } [a,b] = \int_a^b f(x)\,dx = F(b) - F(a)$$

F = some anti-derivative of $f(x)$

Ex

For the example above $f(x) = x^2$, $[0,4]$, $a = 0$, $b = 4$

power rule

Area under graph of $f(x)$ on $[0,4]$ is $\int_a^b f(x)\,dx = \int_0^4 x^2\,dx = \left.\frac{x^3}{3}\right|_0^4 = \frac{4^3}{3} - \frac{0^3}{3} = 21.\overline{3}$

$= \frac{64}{3}$

learn how to evaluate definite integrals, eg ($\int_a^b f(x)\,dx$)

$$\int_0^1 \frac{1}{1+x^2}\,dx = \underbrace{\tan^{-1}x \Big|_0^1}_{\text{find anti-deriv.}} = \tan^{-1}(1) - \tan^{-1}(0) = \underbrace{\text{what angle gives slope} = 1}_{\frac{\pi}{4}} - 0$$

what angle gives slope = 0

$$= \pi/4$$

Same area …

green lune and blue fin.