

Name: \_\_\_\_\_

Exam 3 March 30, 2024

Find the indicated antiderivative.

Show your work

1. Let  $f''(x) = 3x + 2$  and assume  $f'(2) = 11$  and  $f(4) = 1$ . Determine  $f(x)$ .

$$f'(x) = \int f''(x) dx = \int 3x + 2 dx = \frac{3x^2}{2} + 2x + C$$

$$f'(2) = \frac{3 \cdot 2^2}{2} + 2 \cdot 2 + C = 11$$

$$6 + 4 + C = 11 \Rightarrow C = 1$$

$$f(x) = \int \left( \frac{3x^2}{2} + 2x + 1 \right) dx = \frac{3}{3 \cdot 2} x^3 + \frac{2x^2}{2} + x + C$$

$$f(4) = 1 \Rightarrow \frac{1}{2} (4)^3 + 4^2 + 4 + C = 1$$

$$32 + 16 + 4 + C = 1 \Rightarrow C = -51$$

$$\boxed{\frac{1}{2}x^3 + x^2 + x - 51 = f(x)}$$

2.  $\int \frac{3x + 5x^2 + 7}{x} dx =$

can't integral factor by factor

algebra 1st :  $\int \frac{3x}{x} + \frac{5x^2}{x} + \frac{7}{x} dx = \int 3 + 5x + \frac{7}{x} dx = \boxed{3x + \frac{5x^2}{2} + 7 \ln|x| + C}$

3.  $\int x^3 \sqrt{x^5} dx =$

$$\int x^{3 + 5/2} dx = \int x^{11/2} dx = \frac{2}{13} x^{13/2} + C$$

4.  $\int 3x \sin(x^2) dx = \int 3x \cdot \sin(u) \cdot \frac{1}{2x} du = \frac{3}{2} \int \sin(u) du = -\frac{3}{2} \cos(u) + C$

see 1 sin think =  $\int \sin(u) du$

$$\boxed{-\frac{3}{2} \cos(x^2) + C}$$

do.  $\boxed{u = x^2} \quad \begin{cases} du = 2x dx \\ \frac{1}{2x} du = dx \end{cases}$

5.  $\int \frac{5x^4}{\sqrt{x^5-1}} dx = \int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = \frac{u^{1/2}}{1/2} + C = \boxed{2(x^5-1)^{1/2} + C}$

see degree 1 diff  
set  $u =$  higher degree term,  
include constants

$$u = x^5 - 1$$

$$du = 5x^4 dx$$

$$6. \int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx = \int \sin^{-1}(x) \cdot \frac{1}{\sqrt{1-x^2}} dx$$

see deriv. relation

$$u = \sin^{-1}(x)$$

$$du = \frac{1}{\sqrt{1-x^2}} dx$$

$$= \int u \frac{1}{\sqrt{1-x^2}} \sqrt{1-x^2} du = \int u du = \frac{u^2}{2} + C$$

$$\boxed{\frac{(\sin^{-1}(x))^2}{2} + C}$$

$$\sqrt{1-x^2} du = dx$$

$$7. \int \frac{\sec(x)}{e^{\tan(x)} \cos(x)} dx = \int \frac{e^{-\tan x} \cdot \sec(x)}{\cos(x)} dx$$

see 1 e:  $u = \tan x$

$$du = \sec^2 x dx$$

$$\frac{1}{\sec^2 x} du = dx$$

$$u = -\tan x$$

$$w = -u$$

$$dw = -du$$

$$-dw = du$$

$$\Rightarrow \int e^w (-dw) = -\int e^w dw = -e^w + C$$

$$= \boxed{-e^{(-\tan x)} + C}$$

$$8. \int \frac{2x-2}{x^2+1} dx = \int \frac{2x}{x^2+1} - \frac{2}{x^2+1} dx = \int \frac{du}{u} - 2 \int \frac{1}{x^2+1} dx$$

split up:

$$u = x^2 + 1$$

$$du = 2x$$

$$\tan^{-1} x$$

$$\ln|u| \quad \tan^{-1} x$$

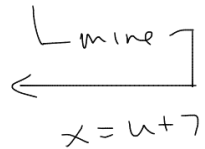
$$\boxed{\ln|x^2+1| - 2 \tan^{-1} x + C}$$

$$9. \int x\sqrt{x-7} dx = \int x\sqrt{u} du = \int (u+7)\sqrt{u} du = \int u^{3/2} + 7u^{1/2} du$$

same degree

$$\text{set } u = x-7$$

$$du = dx$$



$$\text{algebra } 3/2 + 7u^{1/2} du$$

$$= \frac{2}{5} u^{5/2} + \frac{7u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{5} (x-7)^{5/2} + \frac{14}{3} (x-7)^{3/2} + C}$$

deg. 1 diff

$$10. \int 8x^3 \sqrt{x^4-1} dx = 2 \int 4x^3 \sqrt{x^4-1} dx = 2 \int \sqrt{u} du = 2 \int u^{1/2} du$$

$$u = x^4 - 1$$

$$du = 4x^3 dx$$

$$\frac{2 \cdot 2}{3} u^{3/2} + C = \frac{4}{3} (x^4 - 1)^{3/2} + C$$

no deg 1 diff  
sec<sup>-1</sup>

think

$$\int \frac{3}{x \sqrt{x^8-1}} dx = \int \frac{3}{x \sqrt{u^2-1}} \cdot \frac{1}{4x^3} du$$

set  $(u^2 = x^8)$   
 $u = x^4$

$$du = 4x^3 dx$$

$$\frac{1}{4x^3} du = dx$$

$$= \frac{3}{4} \int \frac{1}{x^4 \sqrt{u^2-1}} du = \frac{3}{4} \int \frac{1}{u \sqrt{u^2-1}} du$$

$$= \frac{3}{4} \sec^{-1}(x^4) + C$$

see 1 e

$$u = \sqrt{x}$$

$$du = \frac{1}{2} x^{-1/2} dx$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{e^u}{\sqrt{x}} \cdot 2x^{1/2} du = 2 \int e^u du$$

$$2e^{\sqrt{x}} + C$$

$$\frac{\sin^2 + \cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$

$$2x^{1/2} du = dx$$

$$13. \int \tan(x) \sec^2(x) dx = \int u du = \frac{u^2}{2} + C = \frac{\tan^2 x}{2} + C$$

$$u = \tan x$$

$$du = \sec^2 x$$

or

$$\int \underbrace{\sec(x)}_u \cdot \underbrace{\sec(x) \tan(x)}_{du} = \int u du = \frac{u^2}{2} + C$$

$$= \frac{\sec^2(x)}{2} + C$$

no deriv. rel

$$\int \frac{2x}{\sqrt{1-x^4}} dx = \text{think} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1}(x^2) + C$$

$$u^2 = x^4$$

$$u = x^2, du = 2x dx$$

$$15. \int \frac{\ln(2x)}{x} dx = \int \ln(2x) \cdot \frac{1}{x} dx = \int u du = \frac{u^2}{2} + C$$

$$\ln 2x = \ln 2 + \ln x$$

const.

$$\frac{d}{dx} (\ln 2x) = \frac{1}{x}$$

$\frac{1}{2x}$  →

$$= \frac{\ln(2x)^2}{2} + C$$

$$16. -\int \sin(x) \sqrt{1+\cos(x)} dx = -\int u^{3/2} du = -\frac{2}{5} (1+\cos x)^{5/2} + C$$

$$u = 1 + \cos x$$

$$du = -\sin x$$

$$17. \int \frac{x}{\sqrt{x+4}} dx = \int \frac{u-4}{\sqrt{u}} du = \int \frac{u}{\sqrt{u}} - \frac{4}{\sqrt{u}} du$$

$$u = x+4$$

$$du = dx$$

$$\text{see \# 2} = \int u^{1/2} - 4u^{-1/2} du$$

$$\frac{2}{3} (x+4)^{3/2} - 8(x+4)^{1/2} + C$$

18. Fill in the blank. One thing I think I'll remember from this class is

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