Wk 11 - Mon

Exam 3 - Monday, Nov, 11

Today: u-substitution (technique for integration)

Integration is "harder" than differentiation

$f(x) = \cos(x^2)$

$f'(x) = -\sin(x^2)\cdot 2x$

$\int \cos(x^2)\,dx = ?$ not covered in Calc. I

$g(x) = e^{x^2}$

$g'(x) = e^{x^2}\cdot 2x$

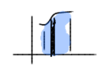
$\int e^{x^2}\,dx =$ there does not exist an "elementary" solution

key: chart! & mind your du's & dx's

$u = f(x)$

Integrals	Ans
$\int u^n\,du = \frac{u^{n+1}}{n+1}$ $(n \neq -1)$	
$\int \frac{1}{u}\,du = \ln\lvert u\rvert$	
$\int \cos(u)\,du = \sin(u)$	
$\int \sin(u)\,du = -\cos(u)$	
$\int \sec^2(u)\,du = \tan(u)$	
$\int \sec(u)\tan(u)\,du = \sec(u)$	
$\int e^u\,du = e^u$	

Example: $\int y^2\,dy = \frac{y^3}{3} + c$



$\int_a^b f(x)\,dx$

height, width

$\int \underbrace{(2x+1)^3}_{\text{is not exactly}}\,dx = ???$

① think: outside u^3, inside $(2x+1)$

set $\boxed{u = 2x+1}$

② apply $\frac{d}{dx}$: $\frac{d}{dx}(u) = \frac{d}{dx}(2x+1)$

$\frac{du}{dx} = 2$

③ isolate dx: $dx\,\frac{du}{dx} = 2\,dx$

$du = 2\,dx$

④ $\boxed{\frac{1}{2}\,du = dx}$

⑤ sub what you can, re-copy what you can't

$= \int (u)^3 \frac{1}{2}\,du$

$= \frac{1}{2}\underbrace{\int u^3\,du}_{\text{matches chart}} = \frac{1}{2}\cdot\frac{u^4}{4} + c$

$= \frac{u^4}{8} + c = \frac{(2x+1)^4}{8} + C$

check: $\frac{d}{dx}(\text{ans}) = \frac{1}{8}\cdot 4(2x+1)^3\cdot 2 = (2x+1)^3$ ☺

"Level 1"

$$\int e^{5x}\,dx$$

① match chart : think $\int e^{u}\,du$

set $\boxed{u = 5x}$

② compute

$\frac{du}{dx} = 5 \Rightarrow du = 5dx \Rightarrow \boxed{\frac{1}{5}du = dx}$

③ sub $= \int e^{u}\frac{1}{5}du = \frac{1}{5}\int e^{u}du = \frac{1}{5}e^{u} + c = \boxed{\frac{1}{5}e^{5x} + c}$

same variable
(only 1 variable!)

Level 2

$$\int \cos(x^2)\,dx = \text{ not a CalcI integral}$$

$\int u^n\,du$

$\int \cos(u)\,du$

$\int x\cos(x^2)\,dx$

$\|$

$\int \cancel{x}\cos(u)\frac{1}{2\cancel{x}}du$

$\|$ algebra

$\frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + c = \boxed{\frac{1}{2}\sin(x^2) + c}$

① think $\int \cos(u)\,du$

$\boxed{u = x^2}$

② $\frac{du}{dx} = 2x$

③ $du = 2x \cdot dx$

④ $\boxed{\frac{1}{2x}du = dx}$

$\frac{d}{dx}\left(\frac{1}{2}\sin(x^2) + c\right)$

$\frac{1}{2}\cos(x^2)\cdot 2x$

$\cos(x^2)\cdot x$

You try:

think: $\int \frac{1}{u}\,du$

$u = 5x+1$ | $du = 5\,dx$

$\frac{du}{dx} = 5$ | $\frac{1}{5}\,du = dx$

$$\int (5x+7)^{-1}\,dx = \int \frac{1}{(5x+1)}\,dx$$

$$= \int \frac{1}{u}\cdot\frac{1}{5}\,du = \frac{1}{5}\int \frac{1}{u}\,du = \frac{1}{5}\ln|u| + c = \frac{1}{5}\ln|5x+7| + c$$

exploit power $\frac{1}{\sqrt{\ }} = (\ )^{-1/2}$

$$\int \frac{x}{\sqrt{x^2+1}}\,dx = \int x(x^2+1)^{-1/2}\,dx$$

think $\int u^n\,du$

$u = x^2+1$

$du = 2x\,dx$ | $dx = \frac{1}{2x}\,du$

$$\int x\cdot u^{-1/2}\,\frac{1}{2x}\,du = \int u^{-1/2}\,\frac{1}{2}\,du = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \boxed{(x^2+1)^{\frac{1}{2}} + c}$$