Wk 11 —— Monday ——

Exam 3 - Next Monday (1 week from today)

u-substitution: (a way of evaluating integrals)
- Integration is hard (more so than differentiation) (- no "rules")

Ex.

$f(x) = -4x^3 + 27x^2 - 42$

$f'(x) = -12x^2 + 54x - 0$

$\int f(x) = \frac{-4x^4}{4} + \frac{27x^3}{3} - 42x + C$

$f(x) = e^{x^2}$

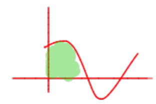
$f'(x) = e^{x^2} \cdot 2x$

$\int f(x)\,dx$ = Doesn't exist as closed form sol'n

Key: revisit chart w/ "u" instead of x ——

Integrals

"what's inside" must match the differential



$\cos(x)$

x

function	Integral
$\int u^n \, du$ $(n \neq -1)$	$\frac{u^n}{n+1}$
$\int \frac{1}{u} \, du$	$\ln\lvert u\rvert$
$\int \cos(u) \, du$	$\sin(u)$
$\int \sin(u) \, du$	$-\cos(u)$
$\int \sec^2(u) \, du$	$\tan(u)$
$\int \sec(u)\tan(u) \, du$	$\sec(u)$
$\int e^u \, du$	e^u

— Basic Example ——— Level 1

Check:

$\frac{d}{dx}\left(\frac{1}{2}\sin(2x)\right) = \frac{1}{2}\cos(2x)\cdot 2 = \cos(2x)$

Ex

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \cos(2x)\,dx$$

now the inside $= 2x$

yet differential is dx | $\rightarrow \int \cos(u)\,du$ doesn't apply

Sol'n: ① identify "inside", set $\boxed{u = 2x}$

② apply $\frac{d}{dx}$: $\frac{d}{dx}(u) = \frac{d}{dx}(2x)$

$\frac{du}{dx} = 2$

③ isolate dx: $dx\left(\frac{du}{dx}\right) = dx(2)$

$du = 2dx$

④ $\boxed{\frac{1}{2}du = dx}$

⑤ substitution $\int \cos(2x)\,dx = \int \cos(u)\,\frac{1}{2}du \overset{\text{pull out}}{=} \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + C$

$= \boxed{\frac{1}{2}\sin(2x) + C}$

Level 2 example

$\int -\tan(4x)+7\,dx$

① look to chart
try to match type:
$\int \tan(u)\,du \rightarrow$ NO

$\Longrightarrow$ doesn't yield to u-sub.

$-\tan(4x)+7$

not isolated $\Rightarrow$ can't remove

$$\int \sec^2(x^3+1)\cdot(x^2)\,dx$$

only sub what you can
– keep what you can't sub

① look to chart: $\int \sec^2(u)\,du$

② match

$\boxed{u = x^3+1}$, $\frac{du}{dx} = 3x^2$, $du = 3x^2\,dx$

isolate dx

$\boxed{\frac{1}{3x^2}du = dx}$

$$\int \sec^2(u)\cdot \cancel{x^2}\cdot\frac{1}{3\cancel{x^2}}\,du$$

|| algebra

$$\int \sec^2(u)\frac{1}{3}\,du = \frac{1}{3}\int \sec^2(u)\,du = \frac{1}{3}\tan(u)+C$$

$$= \frac{1}{3}\tan(x^3+1)+C$$

chain rule

check: $\frac{d}{dx}\left(\frac{1}{3}\tan(x^3+1)\right) = \frac{1}{3}\cdot\sec^2(x^3+1)\cdot 3x^2 = \sec^2(x^3+1)\,x^2$ ☺

EX

$$\int (3x+1)^5\,dx = \int u^5 \tfrac{1}{3}\,du = \tfrac{1}{3}\int u^5\,du$$

$\boxed{u = 3x+1}$

$\frac{du}{dx} = 3$, $du = 3\,dx$, $\boxed{\frac{1}{3}du = dx}$

$$\frac{1}{3}\cdot\frac{u^6}{6} + c$$

$$= \frac{1}{18}\cdot(3x+1)^6 + C$$

Keys:

1. variables must match

$\int 3x+1$ NO sense

$\int 3x+1\,du$ NO sense

$\int u^5\,dx$ NO sense

2. smooth transition from given to your new one

$$\int x\cdot e^{x^2}\,dx = \int \cancel{x} e^u \frac{1}{2\cancel{x}}\,du$$ $=$ algebra saves

① chart? (CIE?) $\Rightarrow \int e^u\,du$

$\boxed{u = x^2}$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\boxed{\frac{1}{2x}du = dx}$

$$\int e^u \tfrac{1}{2}\,du = \tfrac{1}{2}\int e^u\,du$$

$$= \tfrac{1}{2}e^{x^2} + C$$