Wk 11 – Thurs

Exam 3 = Monday

Any Questions

$$\frac{d}{dx}\left(\frac{1}{4}\tan^{-1}(x^4)\right) = \frac{1}{4}\,\frac{1}{1+(x^4)^2}\,4x^3$$

$$= \frac{x^3}{1+x^8}$$

$$\int \frac{x^3}{1+x^8}\,dx \sim \int \frac{1}{1+u^2}\,du$$

want $\boxed{u^2 = x^8}$

so (1) set $\boxed{u = x^4}$

sub $= \displaystyle\int \frac{x^3}{1+u^2}\,\frac{1}{4x^3}\,du$

$$= \int \frac{1}{(1+u^2)4}\,du = \frac{1}{4}\int \frac{1}{(1+u^2)}\,du = \frac{1}{4}\tan^{-1}(u) + C$$

$$= \frac{1}{4}\tan^{-1}(x^4) + C$$

(2) $\frac{du}{dx} = 4x^3$

$du = 4x^3 \cdot dx$

$\boxed{\frac{1}{4x^3}\,du = dx}$

9. $\int \frac{6}{x\sqrt{x^6-1}}\,dx =$

match chart ? — not exactly

degree 1 diff ? no

algebra ? no

$\sim \int \frac{1}{|u|\sqrt{u^2-1}}\,du$ inv. trig

$= \sec^{-1}(u)$

want $\boxed{u^2 = x^6}$

set $\boxed{u = x^3}$

$\frac{du}{dx} = 3x^2$ $\boxed{\frac{1}{3x^2}\,du = dx}$

$du = 3x^2\,dx$

sub $\int \frac{6}{\cancel{x}\sqrt{u^2-1}} \cdot \frac{1}{3\cancel{x^2}}\,du$

$= \int \frac{2}{x^3\sqrt{u^2-1}}\,du \quad \int \frac{2}{|u|\sqrt{u^2-1}}\,du$

$x^3 = u = |u|$

$= 2\sec^{-1}(u) + C$

$\boxed{2\sec^{-1}(x^3) + C}$

check: $d(\text{ans}) = \frac{3(\ln(x))^2}{3} \cdot \frac{1}{x} = (\ln(x))^2 \cdot \frac{1}{x}$ ☺

Antiderivatives 8

Find the indicated antiderivative. Check your answers.

$u = ?$

chart: derivative relationships

set $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

1. $\int \frac{(\ln x)^2}{x}\,dx = \int (\underbrace{\ln x}_{u})^2 \cdot \frac{1}{x}\,dx = \int u^2\,du = \frac{u^3}{3} + c$

this = derivative of

$= \boxed{\frac{(\ln(x))^3}{3} + c}$

$u = ?$

2. $\int \frac{1}{x \ln x}\,dx = \int \frac{1}{x \cdot u} \cdot x\,du = \boxed{\int \frac{du}{u} = \ln|u| + c}$

$\boxed{u = \ln(x)}$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$\boxed{x\,du = dx}$

$\ln(\ln(x)) + C$

ANS

$\frac{d}{dx}(\text{ans})$

$= \frac{1}{\ln(x)} \cdot \frac{1}{x}$

$= \frac{1}{x \ln x}$ ☺

3. $\int \frac{e^{\sec^{-1} x}}{x\sqrt{x^2-1}}\,dx = \int e^{\sec^{-1}(x)} \cdot \frac{1}{x\sqrt{x^2-1}}\,dx = \int e^u \cdot \frac{1}{\cancel{x\sqrt{x^2-1}}}\,\cancel{x\sqrt{x^2-1}}\,du$

$\boxed{u = \sec^{-1}(x)}$

$\frac{du}{dx} = \frac{1}{x\sqrt{x^2-1}}$

$du = \frac{1}{x\sqrt{x^2-1}}\,dx$

$\boxed{x\sqrt{x^2-1}\,du = dx}$

$= \int e^u\,du = e^u + c$

$\boxed{e^{\sec^{-1}(x)} + c}$

4. $\int \frac{2x+1}{x^2+1}\,dx = \int \frac{2x}{x^2+1} + \frac{1}{x^2+1}\,dx$

$u = x^2+1$

$\tan^{-1} x$

5. $\int \frac{\tan^{-1} x}{x^2+1}\,dx = \int \tan^{-1} x \cdot \frac{1}{x^2+1}\,dx = \int u\,du = \frac{u^2}{2} + c$

$u = \tan^{-1} x$

$du = \frac{1}{x^2+1}\,dx$

$\boxed{\frac{(\tan^{-1} x)^2}{2} + c}$

$$\int \frac{1}{\cancel{x}(u)^2} \cdot \cancel{x}\, du = \int \frac{1}{u^2}\, du = \int u^{-2}\, du = \frac{u^{-1}}{-1} + C$$

$$= -(\ln x)^{-1} + C$$

6. $\int \frac{1}{x(\ln x)^2}\, dx = \int \frac{1}{u^2}\, du$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$x\, du = dx$

7. $\int \frac{1}{\left(\sin^{-1} x\right)^3 \sqrt{1-x^2}}\, dx = \int \frac{1}{u^3 \sqrt{1-x^2}} \sqrt{1-x^2}\, du = \int \frac{du}{u^3} = \int u^{-3}\, du$

$$= \frac{u^{-2}}{-2} + C$$

$u = \sin^{-1} x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$du = \frac{dx}{\sqrt{1-x^2}}$

$\boxed{\sqrt{1-x^2}\, du = dx}$

$$\frac{(\sin^{-1} x)^{-2}}{-2} + C$$

8. $\int \frac{\left(\tan^{-1} x\right)^2}{x^2+1}\, dx =$

9. $\int \frac{e^{2x}}{1-e^{2x}}\, dx =$

10. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx =$

$f(x) = \frac{x^3}{2} + x^2 + x - 51$

$f(4) = 1 = \frac{4^3}{2} + 4^2 + 4 + D = 32 + 16 + 4 + D$

$1 = 52 + D$

$D = -51$

Exam 3 Study Guide: Part I

Find the indicated antiderivative.

1. Let $f''(x) = 3x + 2$ and assume $f'(2) = 11$ and $f(4) = 1$. Determine $f(x)$.

$f(x) = \int f'(x)\,dx$; $f'(x) = \int f''(x)\,dx = \int 3x + 2\,dx$

$= \int \frac{3x^2}{2} + 2x + 1\,dx$

$f'(x) = \frac{3x^2}{2} + 2x + C$

$= \frac{3}{2}\frac{x^3}{3} + \frac{2x^2}{2} + x + D = \frac{x^3}{2} + x^2 + x + D$

$f'(2) = 11 = \frac{3 \cdot 2^2}{2} + 2 \cdot 2 + C$

$11 = 6 + 4 + C$

$1 = C$

2. $\int \frac{3x + 5x^2 + 7}{x} dx =$

$\int \frac{3x}{x} + \frac{5x^2}{x} + \frac{7}{x}\,dx = \int 3 + 5x + \frac{7}{x}\,dx$

$= 3x + \frac{5x^2}{2} + 7 \cdot \ln|x| + C$

$x^{5/2}$

3. $\int x^3\sqrt{x^5}dx \neq \frac{x^4}{4} \cdot \int \sqrt{x^5}\,dx$

algebra

$\int x^3 \cdot x^{5/2}\,dx = \int x^{\frac{2}{2}\cdot 3 + 5/2}\,dx = \int x^{11/2}\,dx = \frac{2}{13}x^{\frac{13}{2}} + C$

4. $\int 3x \sin\left(x^2\right)\, dx =$

5. $\int \frac{5x^4}{\sqrt{x^5 - 1}}\, dx =$