Wed - Week 11

5.3.10

$$\int \frac{92}{\sqrt[3]{x}}\,dx \overset{\text{algebra}}{=} 92\int \frac{1}{\sqrt[3]{x}}\,dx = 92\int \frac{1}{x^{1/3}}\,dx \overset{\text{power}}{=} 92\int x^{-\frac{1}{3}}\,dx$$

$$-\frac{1}{3}+\frac{3}{3}=\frac{2}{3}$$

$$= \frac{92x^{-\frac{1}{3}+\frac{3}{3}}}{\frac{2}{3}} = \frac{92x^{\frac{2}{3}}}{\frac{2}{3}} = \frac{3}{2}(92)x^{\frac{2}{3}} = \boxed{148x^{\frac{2}{3}} + c}$$

$$\frac{d}{dx}(\text{ans}) = \frac{2}{3}\cdot\frac{3}{2}\cdot 92x^{\frac{2}{3}-\frac{3}{3}} = \underline{92x^{-1/3}}$$

A.D.3 #5

recall: integrand must match chart exactly!

$$5. \int \frac{4x+5}{x^2+1}\,dx = \int \frac{4x+5}{u}\cdot\frac{1}{2x}\,du = \int \frac{2+\frac{5}{2x}}{u}\,du = \int \underbrace{\frac{2}{u}}_{2\ln|u|} + \underbrace{\frac{\frac{5}{2x}}{u}}_{\text{stuck b/c two variables}}\,du$$

u= higher degree term

$\boxed{u = x^2+1}$

$\frac{du}{dx} = 2x$, $du = 2x\,dx$, $\boxed{\frac{1}{2x}\,du = dx}$

Split it up

$$\underbrace{\int \frac{4x}{x^2+1}\,dx}_{\text{similar to our attempt above}} + \underbrace{\int \frac{5}{x^2+1}\,dx}_{\tan^{-1}x} = \int \frac{4\cancel{x}}{u}\cdot\frac{1}{2\cancel{x}}\,du$$

$u = x^2+1$

$$= \int \frac{2}{u}\,du + 5\tan^{-1}x + c$$

$$= 2\ln|u| + 5\tan^{-1}x + c$$

$$= 2\ln|x^2+1| + 5\tan^{-1}x + c$$

8. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx = \int \sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}\,dx \overset{5}{=} \int u \cdot \frac{1}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\,du = \int u^1\,du$ ← power

$= \frac{u^2}{2} + C$

$= \boxed{\frac{(\sin^{-1}(x))^2}{2} + C}$

derivative relationships

see! $\sin^{-1}x$ AND we see it's derivative $\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow$ (1) set $\boxed{u = \sin^{-1}x}$

(2) $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ (3) $du = \frac{dx}{\sqrt{1-x^2}}$ (4) $\boxed{\sqrt{1-x^2}\,du = dx}$

$3\int \frac{1}{\sqrt{1-x^2}}\,dx =$

$u = 1-x^2$

$du = -2x\,dx$ $\Rightarrow$ stuck

$dx = \frac{-1}{2x}\,du$

$\int \frac{3}{\sqrt{u}} \cdot \frac{-1}{2x}\,du$

two variables :(

$3\sin^{-1}x + C$

10. $\int \frac{x-3}{\sqrt{1-x^2}}\,dx =$

algebra

$\frac{1}{-2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx - \int \frac{3}{\sqrt{1-x^2}}\,dx$

u sub!

$u = 1-x^2$

$du = -2x\,dx$

stuck b/c this isn't exactly a multiple of numerator

$\frac{du}{dx} = -2x$

$\boxed{du = -2x\,dx}$

$-\frac{1}{2x}\,du = dx$

$\frac{-1}{2}\int \frac{du}{\sqrt{u}}$

$-\frac{1}{2}\int u^{-1/2}\,du$

$-\frac{1}{2}\frac{u^{1/2}}{1/2} = -u^{1/2} - 3\sin^{-1}x + C$

$\boxed{-(1-x^2)^{1/2} - 3\sin^{-1}x + C}$

$\frac{d}{dx}(\text{ans}) = -\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) - \frac{3}{\sqrt{1-x^2}}$

:)

Mining -u

① $\int x\sqrt{x}\,dx = \int x \cdot x^{\frac{1}{2}}\,dx = \int x^{\frac{3}{2}}\,dx = \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + c = \frac{2}{5}x^{\frac{5}{2}} + c$

stuck b/c two variables

② $\int x\sqrt{x+1}\,dx = \int x(x+1)^{\frac{1}{2}}\,dx \overset{\text{sub}}{=} \int x(u)^{\frac{1}{2}}\,du \overset{☆}{=} \int (u-1)u^{\frac{1}{2}}\,du$

Strategy: set u = what's inside parenthesis (under $\sqrt{}$)

$u = x + 1$

$\frac{du}{dx} = 1$

$du = dx$

since u = x+1 is a simple equation, we can express x exactly in terms of u

since $u = x+1$

$\Rightarrow u - 1 = x$ ☆

distribute $u^{\frac{1}{2}}$

$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du = \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$

Check $\frac{d}{dx}(\text{ans}) = \frac{5}{2}\cdot\frac{2}{5}(x+1)^{\frac{3}{2}} - \frac{3}{2}\cdot\frac{2}{3}(x+1)^{\frac{1}{2}} = (x+1)^{\frac{3}{2}} - (x+1)^{\frac{1}{2}}$

$= (x+1)^{\frac{1}{2}}\left[(x+1)^{3/2-1/2} - (x+1)^{\overbrace{\frac{1}{2}-\frac{1}{2}}^{=0}}\right]$

$= (x+1)^{\frac{1}{2}}\left[x+1-1\right] = (x+1)^{\frac{1}{2}}\cdot x$