Antiderivatives 4

Find the indicated antiderivative. Check your answers.

1. $\int 5x^2 \cos\left(x^3\right)\, dx =$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\, dx$

$\frac{1}{3x^2} du = dx$

$\int 5x^2 \cos(u) \frac{1}{3x^2} du = \int \frac{5}{3} \cos(u)\, du$

$= \frac{5}{3} \int \cos(u)\, du = \frac{5}{3} \sin(u) + C$ (chart)

$= \boxed{\frac{5}{3} \sin\left(x^3\right) + C}$

2. $\int 2x \sin\left(x^2+1\right)\, dx =$

$u = x^2+1$

$du = 2x\, dx$

$\int \sin(u)\, du$

$\frac{1}{2} \int \sin\left(x^2+1\right) \cdot 2x\, dx$ (du)

$\boxed{-\frac{1}{2} \cos\left(x^2+1\right) + C}$

3. $\int 2x^3 \sec\left(x^4-1\right) \tan\left(x^4-1\right)\, dx =$

4. $\int 3x^2 \sec^2\left(x^3\right)\, dx =$

5. $\int \csc^2(5x)\, dx =$

— Strategies for Integrals —

1. easy, no u-sub (chart)
2. u-sub:
   (i) deriv. relationships
   eg. (degree 1 differences)
   (a) u = higher degree term (include constants)
   (b) $\sin(x) \overset{\int}{\underset{d}{\leftrightarrows}} \cos(x)$
   (c) $e^x \overset{\int}{\underset{d}{\leftrightarrows}} e^x + 1$
   (d) $\tan(x) \overset{\int}{\underset{d}{\leftrightarrows}} \sec^2(x)$
   (ii) stuck? try: set u = inside (1)
   set u = more complicated
3. if u-sub fails →
   (i) algebra to simplify
   (ii) inverse trig derivative

6. $\int e^{3x-5}\,dx =$

7. $\int 2x^2\,e^{x^3}\,dx =$

check $\frac{d}{dx}(eu) = e^{x^2+2x+1}(2x+2)$ ☺

$2(x+1)$

C1E

8. $\int (x+1)\,e^{x^2+2x+1}\,dx = \int e^u\,du$

$u = x^2+2x+1$

$\frac{du}{dx} = 2x+2$

$du = (2x+2)\,dx$

$\frac{1}{2x+2}\,du = dx$

$= \int (x+1)\,e^u \left(\frac{1}{2x+2}\right) du$

$2(x+1)$

$\int \frac{1}{2}e^u\,du = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^{x^2+2x+1} + C$

9. $\int 4\cos x\,e^{\sin x}\,dx = \int e^u\,du$

$4\int \cos x\,e^{\sin x}\,dx$

$4\int e^u\,du = 4e^{\sin x} + C$

$u = \sin x$

$du = \cos x\,dx$

multiply by dx

$\frac{d}{dx}(u) = \frac{du}{dx} = \cos x$

$\frac{d}{dx}(eu) = 4e^{\sin x}\cdot\cos x$ ☺

10. $\int 2\sec^2 x\,e^{\tan x}\,dx =$

Mining - u

$x(x+1)^{\frac{1}{2}}$ stuck

$$\int x\sqrt{x+1}\,dx$$

① $\int x(x+1)^{\frac{1}{2}}\,dx$

Set u = inside of parenthesis

$u = x+1$ ④ $u-1 = x$

$du = dx$

$\frac{du}{dx} = 1$ ②

$= \int x \cdot u^{\frac{1}{2}}\,du$ ③

stuck b/c: two variables (fix: maybe replace x with something involving u.

⑤ $= \int (u-1)u^{\frac{1}{2}}\,du = \int u^{3/2} - u^{1/2}\,du$ ⑥ algebra

$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$

wow — what a difference adding one makes

$$\int x\sqrt{x}\,dx = \int x \cdot x^{\frac{1}{2}}\,dx$$

$$= \int x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$\frac{2}{5}x^{5/2} + C$

Antiderivatives 5

Find the indicated antiderivative. Check your answers.

1. $\int \frac{2x}{x^2+1}\,dx =$ $\int \frac{2x}{u}\,\frac{1}{2x}\,du = \int \frac{1}{u}\,du = \int \frac{du}{u} = \ln|u| + C$

$= \ln|x^2+1| + C$

$u = x^2+1$ | $du = 2x\,dx$

$\frac{du}{dx} = 2x$ | $\frac{1}{2x}\,du = dx$

2. $\int \frac{\cos x}{1+\sin x}\,dx =$ $\int \frac{\cos x\,dx}{1+\sin x} = \int \frac{du}{u} = \ln|1+\sin x| + C$

$u = 1+\sin x$

$du = \cos x\,dx$

3. $\frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx =$ $\frac{1}{4}\int \frac{du}{u} =$ $\frac{1}{4}\ln|1+x^4| + C$

$4x^3$

$u = 1+x^4$, $du = 4x^3\,dx$

4. $\int \frac{e^x+2}{e^x+2x+7}\,dx =$

5. $\int \frac{3}{2x+5}\,dx =$