6-2 HW

9 questions

Course Info | Instructor Name | Student Name

Question 1 of 9

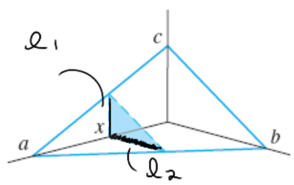
Find the volume of the wedge in the figure by integrating the area of vertical cross sections.

Assume that $a = 6$, $b = 4$, and $c = 3$.

Strategy: Compute area of cross-section:

$A = \frac{1}{2}\text{base} \times \text{height}$

Integrate wrt x.



Key: use similar Δ's to find base & height

(Give an exact answer. Use symbolic notation and fractions where needed.)

$V =$

Let base $= \ell_1$, height $= \ell_2$

Similar Δ's in ac-plane: $\frac{c}{\ell_1} = \frac{a}{a-x}$ & in ba-plane $\frac{b}{\ell_2} = \frac{a}{a-x}$

$\frac{3}{\ell_1} = \frac{6}{6-x}$ & $\frac{4}{\ell_2} = \frac{6}{6-x}$

$6\ell_1 = 3(6-x)$ $\quad$ $6\ell_2 = 4(6-x)$

$\boxed{\ell_1 = \frac{1}{2}(6-x)}$ $\quad$ $\boxed{\ell_2 = \frac{2}{3}(6-x)}$

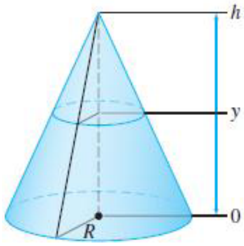
Area of Slice: $\frac{1}{2}\ell_1 \cdot \ell_2 = \frac{1}{2} \cdot \frac{1}{2}(6-x) \cdot \frac{2}{3}(6-x) = \frac{1}{6}(6-x)^2$

$$\int_0^6 \frac{1}{6}(6-x)^2\,dx = \frac{1}{6}\int_0^6 36 - 12x + x^2\,dx = \frac{1}{6}\left[36x - 6x^2 + \frac{x^3}{3}\right]\Big|_0^6$$

$$= \frac{1}{6}\left[6^3 - 6^3 + \frac{6^3}{3}\right] = \frac{1}{6}\left[\frac{6^3}{3}\right] = \frac{6^2}{3} = 12 \text{ cubic units}$$

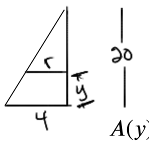
Question 2 of 9

Let V be the volume of a right circular cone of height $h = 20$ whose base is a circle of radius $R = 4$.



(a) Use similar triangles to find the area of a horizontal cross section at a height y. Give your answer in terms of y.

(Use symbolic notation and fractions where needed.)



$\Rightarrow \frac{20}{4} = \frac{20-y}{r} \Rightarrow 5r = 20 - y \Rightarrow A = \pi r^2 = \pi\left(4 - \frac{y}{5}\right)^2$

$r = 4 - \frac{1}{5}(y)$

$A(y) =$

(b) Calculate V by integrating the cross-sectional area.

(Use symbolic notation and fractions where needed.)

$$\int_0^{20} \pi\left(4-\frac{y}{5}\right)^2 dy = \pi\int_0^{20} 16 - \frac{8y}{5} + \frac{y^2}{25}\, dy = \pi\left[16y - \frac{8y^2}{10} + \frac{y^3}{75}\right]_0^{20}$$

$V =$

$$= \pi\left[16(20) - \frac{8(20)^2}{10} + \frac{20^3}{75}\right] = \pi \cdot 20\left[16 - \frac{8 \cdot 20}{10} + \frac{20^2}{75}\right] = 20\pi\left[\frac{20^2}{75}\right]$$

$$= \frac{20^3\pi}{75} = \frac{\cancel{20}^{4.8} \cdot \cancel{20}^{4.8}\ 20\ \pi}{\cancel{25} \cdot 3} = \frac{16 \cdot 20\pi}{3} = \frac{320\pi}{3}$$

$\frac{64}{5}$

320

or u-sub

$u = 4 - \frac{y}{5}$

$du = -\frac{1}{5} dy$

$-5du = dy$

$$= \int \pi (u)^2 (-5)\, du = -5\int_4^0 \pi u^2 du = -\frac{5\pi \cdot u^3}{3}\Big|_4^0 = -\frac{5\pi 0^3}{3} - \left(-\frac{5\pi \cdot 4^3}{3}\right)$$

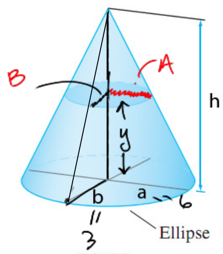
Bounds: $y = 0 \Rightarrow u = 4$

$y = 20 \Rightarrow u = 0$

$= \frac{320 \cdot \pi}{3}$

Question 3 of 9

The area of an ellipse is πab, where a and b are the lengths of the semimajor and semiminor axes. Compute the volume of a cone of height $h = 18$ whose base is an ellipse with semimajor axis $a = 6$ and semiminor axis $b = 3$.



FIGURE

Set y = height of slice

I $\quad \frac{18}{3} = \frac{18-y}{B}$ $\qquad$ II $\quad \frac{18}{6} = \frac{18-y}{A}$

$6B = 18-y$ $\qquad$ $3A = 18-y$

$$\boxed{B = \frac{1}{6}[18-y] \qquad A = \frac{1}{3}[18-y]}$$

Area $= \pi AB$

$= \frac{\pi}{8}[18-y]^2$

$V = \int_0^{18} \frac{\pi}{8}[18-y]^2\,dy$

$u = 18-y$, $du = -dy$

$y = 0 \Rightarrow u = 18$

$y = 18 \Rightarrow u = 0$

(Use symbolic notation and fractions where needed.)

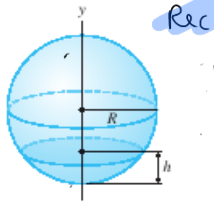
$V =$

$$V = -\frac{\pi}{8}\int_{18}^{0} u^2\,du = -\frac{\pi}{8}\cdot\frac{u^3}{3}\Big|_{18}^{0} = 0 - \left(\frac{-\pi}{24}\cdot\frac{18^3}{3}\right) = \frac{\pi\cdot\overset{6\cdot3}{18}\cdot\overset{2\cdot9}{18}\cdot\overset{2\cdot3\cdot3}{18}}{\underset{6\cdot4}{24}\cdot3}$$

$$= \boxed{81\pi}$$

Question 4 of 9 Start: Circle: $x^2 + y^2 = R^2 \implies x = \pm\sqrt{R^2 - y^2}$

Find the volume of liquid needed to fill a sphere of radius R to height $h = \frac{R}{4}$.



Recall: To find vol. of sphere w/ radius R:

$$V = \int_{-R}^{R} \pi r^2 \, dy \quad \text{with} \quad r = \sqrt{R^2 - y^2}$$

$$= \int_{-R}^{R} \pi (R^2 - y^2)\, dy = \pi \left[R^2 y - \frac{y^3}{3} \right]_{-R}^{R} = \pi \left[R^2 \cdot R - \frac{R^3}{3} - \left(-R^3 + \frac{R^3}{3} \right) \right]$$

$$= \pi \left[R^3 + R^3 - \frac{R^3}{3} - \frac{R^3}{3} \right]$$

$$= \pi \left[2R^3 - \frac{2R^3}{3} \right] = \frac{4\pi R^3}{3}$$

(Use symbolic notation and fractions where needed.)

$V =$

Since $h = 2R \sim$ full sphere

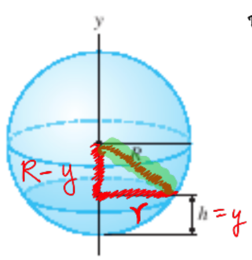
$h = \frac{R}{4}$ is $1/8$ of sphere since $\frac{1}{8}(2R) = \frac{R}{4}$

$[-R, R]$ = full
$[-R, 0]$ = half
$[-R, -\frac{1}{2}R]$ = quarter
$[-R, -\frac{1}{4}R]$ = eighth

Modify bounds: instead of $[-R, R]$ do $[-R, -\frac{1}{4}R]$

$$\int_{-R}^{-R/4} \pi (R^2 - y^2)\, dy = \pi \left[R^2 y - \frac{y^3}{3} \right] \Bigg|_{-R}^{-R/4} = \pi \left[-\frac{R^3}{4} + \frac{R^3}{192} + R^3 - \frac{R^3}{3} \right]$$

$$= \pi \left[\frac{5R^3}{12} + \frac{R^3}{192} \right] = \pi R^3 \left[\frac{5}{12} + \frac{1}{192} \right]$$



Find r of slice:

$$(R - y)^2 + r^2 = R^2$$

$$r = \sqrt{R^2 - (R-y)^2} = \sqrt{R^2 - R^2 + 2Ry - y^2}$$

$$r = \sqrt{y(2R - y)}$$

Hypotenuse is a radius of sphere.

$(R-y)^2 + r^2 = R^2$

$$A = \pi r^2 = \pi \left(\sqrt{2Ry - y^2} \right)^2 = \pi (2Ry - y^2)$$

$$V = \int_0^{R/4} \pi (2Ry - y^2)\, dy = \pi \left(Ry^2 - \frac{y^3}{3} \right) \Bigg|_0^{R/4} = \pi \left(\frac{R^3}{16} - \frac{R^3}{192} \right) = \pi R^3 \left(\frac{15}{192} \right)$$

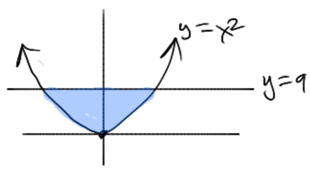
$64 \cdot 3 = 16 \cdot 4 \cdot 3$

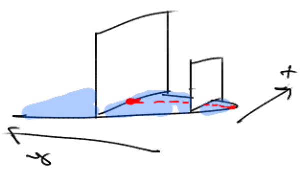
Question 5 of 9

Find the volume of the solid whose base is the region enclosed by $y = x^2$ and $y = 9$, and whose cross sections perpendicular to the y-axis are squares.

(Give your answer as a whole or exact number.)

$V =$



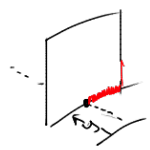


① Int. wrt y

② Bounds $y=0$, $y=9$

③ $y=x^2 \Rightarrow x=\sqrt{y}$

$V = \int$ area of slice



Half edge has length $= \sqrt{y}$

$\Rightarrow$ Area $= (2\sqrt{y})^2 = 4y$

④ $V = \int_0^9 4y\, dy = \frac{4y^2}{2}\Big|_0^9 = 2\cdot(9)^2 = 162$

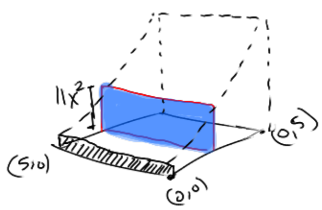
Question 6 of 9

The base of the solid is a square, one of whose sides is the interval $[0, 5]$ along the the x-axis.

The cross sections perpendicular to the x-axis are rectangles of height $f(x) = 11x^2$. Compute the volume of the solid.

(Use symbolic notation and fractions where needed.)

$V =$



$$V = \int_0^5 11x^2 \cdot 5\,dx = 55\int_0^5 x^2\,dx = 55\frac{x^3}{3}\Big|_0^5$$

$$= \frac{55 \cdot 5^3}{3}$$

Question 7 of 9

Find the volume of the solid whose base is the semicircle $y = \sqrt{4 - x^2}$ where $-2 \le x \le 2$ and the cross sections perpendicular to the x-axis are squares.

(Use symbolic notation and fractions where needed.)

$V =$

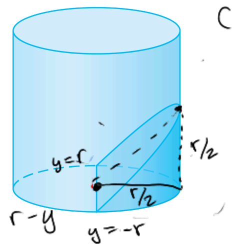
length $= y = \sqrt{4-x^2}$

$A = y^2 = 4 - x^2$

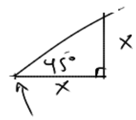
$$\Rightarrow V = \int_{-2}^{2} 4 - x^2 \, dx \overset{\text{even}}{=} 2\int_0^2 4 - x^2 \, dx = 2\left[4x - \frac{x^3}{3}\Big|_0^2\right]$$

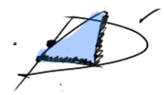
$$= 2\left(8 - \frac{8}{3}\right) = 2\left(\frac{16}{3}\right) = \frac{32}{3}$$

A plane inclined at an angle of 45° passes through a diameter of the base of a cylinder of radius r.



Cross Sections = triangles





$x = \sqrt{r^2 - y^2}$

Write an expression for the volume V of the region within the cylinder and below the plane in terms of r.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$A = \frac{1}{2}x^2 = \frac{1}{2}(r^2 - y^2)$ $\quad V = \int_{-r}^{r} \frac{1}{2}(r^2-y^2)\,dy = \int_0^r (r^2-y^2)\,dy =$

$V(r) =$

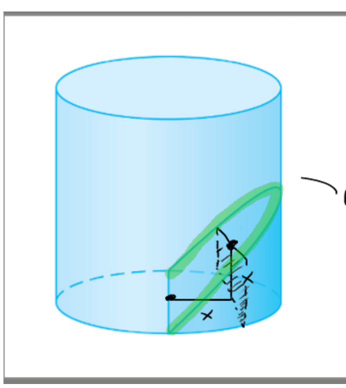
$r^2 y - \frac{y^3}{3}\Big|_0^r = r^3 - \frac{r^3}{3} = \frac{2}{3}r^3$

Calculate the volume of the region within the cylinder and below the plane for $r = 7$.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

OR

$V =$



$R = \sqrt{r^2 - x^2}$

Area, rectangle: $2\sqrt{r^2-x^2} \cdot x$

$\sqrt{r^2-x^2}$ $\quad x$

$\int_0^r 2x\sqrt{r^2-x^2}\,dx$ $\quad u = r^2 - x^2$

$x = 0, u = r^2$

$x = r, u = 0$

$= -\int_{r^2}^{0} \sqrt{u}\,du$

$= \int_0^{r^2} \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2}\Big|_0^{r^2}$

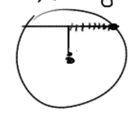
$= \frac{2}{3} \cdot r^3$

Green Arc is a semi-circle

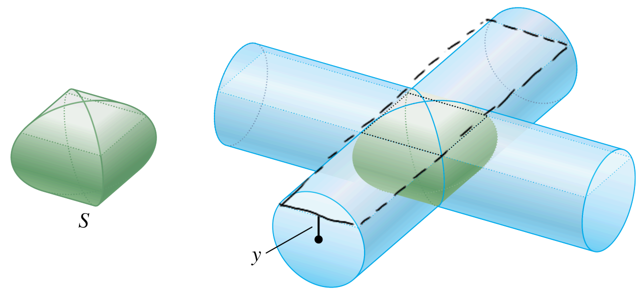
Question 9 of 9

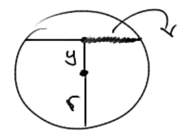
The solid S in the figure is the intersection of two cylinders of radius r whose axes are perpendicular.

$x^2+y^2=1$



$x=\sqrt{1-y^2}$





$\sqrt{r^2-y^2}$

$\Rightarrow$ width $= 2\sqrt{r^2-y^2}$

(a) The horizontal cross section of each cylinder at distance y from the central axis is a rectangular strip. Find the strip's width.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

strip's width:

(b) Find the area of the horizontal cross section of S at distance y.

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$A = 2\sqrt{r^2-y^2}\cdot 2\sqrt{r^2-y^2}$

$= 4(r^2-y^2)$

area:

even

$V = \int_{-4}^{4} 4(r^2-y^2)\,dy = 2\cdot 4\left(r^2 y - \frac{y^3}{3}\right)\Big|_0^4 = 8\left(64-\frac{64}{3}\right) = 2^3\cdot 2^6\left(1-\frac{1}{3}\right)$

(c) Find the volume of S for $r = 4$.

$= 2^9\left(\frac{2}{3}\right)$

(Express numbers in exact form. Use symbolic notation and fractions where needed.)

$= \frac{2^{10}}{3} = \frac{1024}{3}$

=

volume:

$r=7$

$\Rightarrow 2\left(4\cdot 49\cdot 4 - \frac{4^3}{3}\right) = 2\cdot 16\left(49-\frac{4}{3}\right) = 32(47.\bar{6})$