## Exam 3 Study Guide: Part I

Find the indicated antiderivative.

1. Let $f''(x) = 3x + 2$ and assume $f'(2) = 11$ and $f(4) = 1$. Determine $f(x)$.

$f'(x) = \int 3x+2 \, dx = \frac{3x^2}{2} + 2x + C$

$f'(2) = \frac{3(2)^2}{2} + 2(2) + C = 11$

$3 \cdot 2 + 4 + C = 11$

$C = 1$

$f'(x) = \frac{3x^2}{2} + 2x + 1$

$f(x) = \int f'(x) = \frac{x^3}{2} + x^2 + x + D$

$f(4) = \frac{4^3}{2} + 4^2 + 4 + D = 32 + 16 + 4 + D = 52 + D$

$1$

$D = -51$

$\boxed{f(x) = \frac{x^3}{2} + 2x + x - 51}$

2. $\int \frac{3x + 5x^2 + 7}{x} dx =$

$\int 3 + 5x + \frac{7}{x} \, dx = \boxed{3x + \frac{5x^2}{2} + \ln|x| + C}$

3. $\int x^3 \sqrt{x^5} dx =$

$\int x^3 x^{5/2} dx = \int x^{11/2} dx = \boxed{\frac{2}{13} x^{13/2} + C}$

4. $\int 3x \sin(x^2) \, dx = 3\int x \cdot \sin(u) \frac{1}{2x} du = \frac{3}{2}\int \sin(u) \, du = -\frac{3}{2}\cos(u) + C$

$u = x^2$

$du = 2x \, dx$

$\frac{1}{2x} du = dx$

$= \boxed{-\frac{3}{2}\cos(x^2) + C}$

5. $\int \frac{5x^4}{\sqrt{x^5 - 1}} dx = \int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = \frac{u^{1/2}}{(1/2)} + C = \boxed{2(x^5 - 1)^{1/2} + C}$

$u = x^5 - 1$

$du = 5x^4 dx$

Omit any one of the following: ____________________

6. $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(\sin^{-1}(x))^2}{2} + C}$

$u = \sin^{-1}(x)$

$du = \frac{1}{\sqrt{1-x^2}}dx$

7. $\int \frac{\sec(x)}{e^{\tan(x)}\cos(x)}\,dx = \int e^{-\tan x} \cdot \sec(x) \cdot \frac{1}{\cos(x)} dx$

$= \int e^{-\tan x} \cdot \sec^2(x)\,dx$

$u = -\tan x$

$du = -\sec^2(x)\,dx$

$= -\int e^u(-\sec^2(x))\,dx$

$= -\int e^u du = -e^u + C = \boxed{-e^{-\tan(x)} + C}$

8. $\int \frac{2x-2}{x^2+1}\,dx =$

$= \underbrace{\int \frac{2x}{x^2+1}dx}_{u=x^2+1} - \int \frac{2}{x^2+1}dx = \int \frac{du}{u} - 2\int \frac{1}{x^2+1}dx$

$du = 2x\,dx$

$= \ln|u| - 2\tan^{-1}(x) + C$

$= \boxed{\ln|x^2+1| - 2\tan^{-1}(x) + C}$

9. $\int x\sqrt{x-7}\,dx = \int (u+7)\sqrt{u}\,du = \int u^{3/2} + 7\sqrt{u}\,du$

$\left.\begin{array}{l} u = x-7 \\ du = dx \end{array}\right\}$ $u+7 = x$

$= \frac{2}{5}u^{5/2} + \frac{7u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{5}(x-7)^{5/2} + \frac{14}{3}(x-7)^{3/2} + C}$

Omit any one of the following: ____________________

10. $\int 8x^3\sqrt{x^4-1}\,dx = 2\int 4x^3\sqrt{u}\;dx = 2\int \sqrt{u}\,4x^3\,dx = 2\int\sqrt{u}\,du$

$u = x^4-1$

$du = 4x^3\,dx$

$= \dfrac{2u^{3/2}}{3/2} + c = \boxed{\frac{4}{3}\cdot(x^4-1)^{3/2} + C}$

11. $\int \frac{3}{x\sqrt{x^{10}-1}}\,dx = 3\int \frac{1}{x\sqrt{u^2-1}}\,dx = 3\int \frac{1}{x\sqrt{u^2-1}}\cdot\frac{1}{5x^4}\,du$

$u = x^5,\ u^2 = x^{10}$

$du = 5x^4\,dx$

$\frac{1}{5x^4}\,du = dx$

$= \frac{3}{5}\int \frac{1}{x^5\sqrt{u^2-1}}\,du$

$= \frac{3}{5}\int \frac{1}{u\sqrt{u^2-1}}\,du = \frac{3}{5}\sec^{-1}(u) + C$

$= \boxed{\frac{3}{5}\sec^{-1}(x^5) + C}$

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12. $\cancel{\int \frac{\sqrt{x}}{e^{\sqrt{x}}}\,dx} = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\sqrt{x}\,du = dx$

$= \int \frac{e^u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\int e^u\,du = 2e^u + C$

$= \boxed{2e^{\sqrt{x}} + C}$

13. $\int \tan(x)\sec^2(x)\,dx = \int u\,du = \frac{u^2}{2} + c = \boxed{\frac{(\tan(x))^2}{2} + C}$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

Omit any one of the following: ______________________

$u^2 = x^4$

14. $\int \frac{x}{\sqrt{1-x^4}}\,dx = \int \frac{x}{\sqrt{1-u^2}}\,dx = \int \frac{x}{\sqrt{1-u^2}} \cdot \frac{1}{2x}\,du$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$= \int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C$

$= \boxed{\sin^{-1}(x^2) + C}$

15. $\int \frac{\ln(2x)}{x}\,dx = \int \frac{u}{x} \cdot x\,du = \int u\,du = \frac{u^2}{2} + C$

$u = \ln(2x)$

$du = \frac{1}{2x} \cdot 2\,dx = \frac{1}{x}\,dx$

$= \boxed{\frac{(\ln(2x))^2}{2} + C}$

$\boxed{x\,du = dx}$

16. $\int \sin(x)\sqrt{1+\cos(x)}\,dx = \int \sin(x)\sqrt{u} \cdot \left(\frac{1}{-\sin(x)}\right) du$

$u = 1 + \cos(x)$

$du = -\sin(x)\,dx$

$\frac{1}{-\sin(x)}\,du = dx$

$= -\int \sqrt{u}\,du = \frac{-u^{3/2}}{3/2} + C$

$= \boxed{-\frac{2}{3}(1+\cos(x))^{3/2} + C}$

17. $\int \frac{x}{\sqrt{x+4}}\,dx = \int \frac{x}{\sqrt{u}}\,du = \int \frac{u-4}{\sqrt{u}}\,du = \int u^{1/2} - 4u^{-1/2}\,du$

$u = x + 4$ | $u - 4 = x$

$du = dx$

$\frac{2}{3}u^{3/2} - \frac{4u^{1/2}}{1/2} + C$

$= \boxed{\frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + C}$