Limits:

$$\lim_{x\to+\infty} \frac{3x^2+4x+1}{6x^2+170} = \frac{1}{2}$$

old way

when x is large, only leading terms matter!

$$= \lim_{x\to\infty} \frac{3x^2}{6x^2} = \frac{3}{6} = \frac{1}{2}$$

L'Hospital's Rule

Idea:

if $\lim\limits_{x\to+\infty} \dfrac{f(x)}{g(x)} = \dfrac{\infty}{\infty}$ then $= \lim\limits_{x\to\infty} \dfrac{f'(x)}{g'(x)}$

rate of change of f

indeterminate forms #1

Ex.

marches to ∞ much faster than x.

$$\lim_{x\to+\infty} \frac{e^x}{x} = \frac{e^\infty}{\infty} = \frac{\infty}{\infty}$$ Interd. Forms $\Rightarrow$ L'Hopital Applies

$$L = \lim_{x\to+\infty} \frac{e^x}{1} = e^\infty = \infty$$

Also apply L'Hospital's Rule to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x \to 5} \frac{x-5}{x^2-25} = \frac{0}{0}$$

or

$$= \lim_{x \to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(x+5)}$$

$$= \lim_{x \to 5} \frac{1}{x+5} = \frac{1}{10}$$

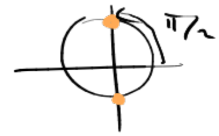
$$\Big\downarrow \text{L'H}$$

$$\lim_{x \to 5} \frac{1}{2x} = \frac{1}{10}$$

Ex

$$\lim_{x \to 0} \frac{\cos^{-1}(x) - \frac{\pi}{2}}{x}$$

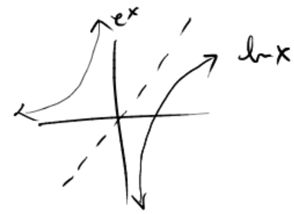
$\cos^{-1}(x)$ = angle required so that cosine = x.



1. plug in limiting x : $\frac{\cos^{-1}(0) - \frac{\pi}{2}}{0} = \frac{\frac{\pi}{2} - \frac{\pi}{2}}{0} = \frac{0}{0}$ Apply L'Hospital

2. $\lim_{x \to 0} \frac{\boxed{\frac{-1}{\sqrt{1-x^2}}}}{1} = \frac{-1}{\sqrt{1-0^2}} = -1$

Ex. $\lim_{x \to +\infty} \frac{(\ln(2x))^2}{2x} = \frac{(\ln(2 \cdot \infty))^2}{2 \cdot \infty} = \frac{(\ln(\infty))^2}{\infty} = \frac{\infty}{\infty}$



L'H

$\lim_{x \to +\infty} \frac{\cancel{2}(\ln(2x))^1 \cdot \left(\frac{\cancel{2}}{\cancel{2}x}\right)}{\cancel{2}}$

$= \lim_{x \to +\infty} \frac{\ln(2x)}{x} = \frac{\infty}{\infty} \xrightarrow{L'H} = \lim_{x \to +\infty} \frac{\left(\frac{2}{2x}\right)}{1} = \lim_{x \to +\infty} \frac{1}{x} = 0$

Ex. $\lim_{x \to +\infty} \frac{(\ln(x))^{14}}{x} \xrightarrow{14 \text{ L'H}} \rightarrow 0$

Ex.

$y = x^{3x} \longrightarrow \ln y = \ln x^{3x} = 3x \cdot \ln x$

$y' = \_\_\_\_$

$\frac{y'}{y} = 3 \cdot \ln x + 3x \cdot \frac{1}{x}$

$y' = (3 \cdot \ln x + 3) \cdot (x^{3x})$

let

$$y = \lim_{x \to +\infty} \left(1 + \frac{3}{x}\right)^{x}$$

goal: find y.

$$\ln y = \ln\left(\lim_{x\to+\infty}\left(1+\frac{3}{x}\right)^{x}\right)$$

continuous functions pass across limits

$e^{\lim(f)} = \lim(e^{f})$

② $\ln(\lim f) = \lim(\ln f)$

$$= \lim_{x\to+\infty} \ln\left(1+\frac{3}{x}\right)^{x}$$

$x = \frac{1}{\left(\frac{1}{x}\right)}$

$$= \lim_{x\to+\infty} x \cdot \ln\left(1+\frac{3}{x}\right) = \infty \cdot \ln\left(1+\frac{3}{\infty}\right) = \infty \cdot \ln(1+0)$$

$$= \infty \cdot \ln(1) = \infty \cdot 0$$

IDK!

$$= \lim_{x\to+\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{\left(\frac{1}{x}\right)} = \frac{\ln(1)}{\frac{1}{\infty}} = \frac{0}{0}$$

Apply L'H

$$= \lim_{x\to+\infty} \frac{\left[\frac{-\frac{3}{x^2}}{1+\frac{3}{x}}\right]}{\frac{-1}{x^2}} = \lim_{x\to+\infty} \left[\frac{\cancel{-}\frac{3}{\cancel{x^2}}}{1+\frac{3}{x}}\right] \cdot \frac{\cancel{-}\cancel{x^2}}{1}$$

$$= \lim_{x\to+\infty} \frac{3}{1+\frac{3}{x}} = \frac{3}{1+\frac{3}{\infty}} = \frac{3}{1+0} = 3$$

$$\left(1 + \frac{3}{\infty}\right)^{\infty} \neq 1$$

$$\frac{3}{\infty} = 0$$

$$\lim_{x \to \infty} \frac{3}{x} = \text{approaches } 0.$$

$$\frac{3}{x} \neq 0$$

$$1^{\infty} = 1 \cdot 1 \cdot 1 \cdot \ldots \infty.$$

$$(1 + .0000001)^{100000000} \to 1$$

$$3$$