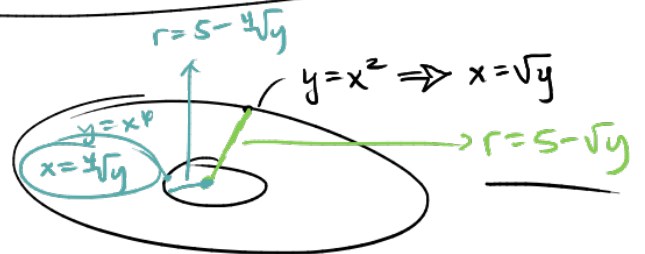


1. slice \perp axis of rotation

2. Here, size of slice depends on y
 (\Rightarrow) get radius as function of y

(always the case when axis of rotation is y -axis or \parallel to it!)



3. Outer Area: $\pi(r)^2 = \pi(5-\sqrt{y})^2$

Inner Area: $\pi r^2 = \pi(5-4\sqrt{y})^2$

$$\left(- (25 - 10\sqrt{y} + y) \right)$$

4. $Vol = \int_0^1 \pi(5-\sqrt{y})^2 - \pi(5-4\sqrt{y})^2 dy$

5. $\pi \int_0^1 25 - 10\sqrt{y} + y - 25 + 10\sqrt[4]{y} - \sqrt{y} dy$

$$\pi \int_0^1 -10\sqrt{y} + y + 10\sqrt[4]{y} - \sqrt{y} = \pi \left(-\frac{20}{3} y^{\frac{3}{2}} + \frac{y^2}{2} + \frac{40}{5} y^{\frac{5}{4}} - \frac{2}{3} y^{\frac{3}{2}} \right) \Big|_0^1$$

$$= \pi \left(-\frac{20}{3} + \frac{1}{2} + 8 - \frac{2}{3} \right) = \pi(1.1\bar{6}) \approx 4$$

$$\begin{array}{r} -\frac{20}{3} = -7.\bar{3} \\ + 8 \\ \hline .6 \\ \hline 1.5 \end{array}$$

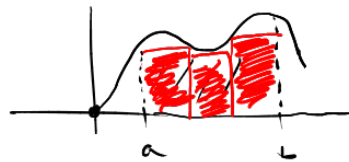
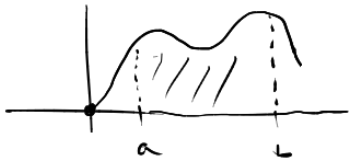
Average Value:

The average value of a ^{continuous} function is similar to the average value of a finite list of numbers.

i.e., $\{2, 3, 7, 4, 5\}$

$$\xrightarrow[\text{Value}]{\text{Average}} \frac{21}{5} = 4.2$$

$$\frac{2+3+7+4+5}{\# \text{ items}} \rightarrow \frac{\int_a^b f(x) dx}{b-a}$$



Recall: Area:

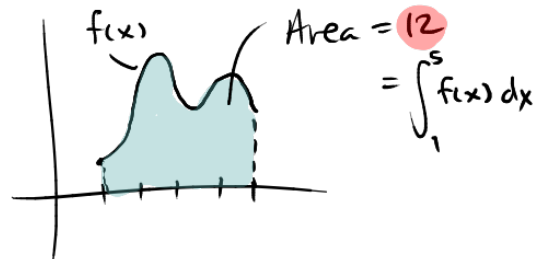
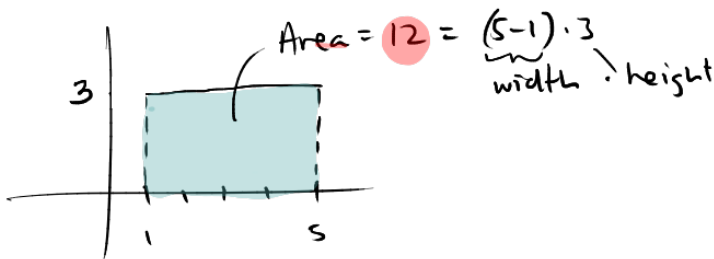
$$\text{Precise Area} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \underbrace{f(x_i)}_{\text{heights}} \underbrace{\Delta x}_{\text{widths}}$$

↑
increase n

↑
How many rectangles

$$= \int_a^b f(x) dx$$

infinite addition



$$12 = (5-1) \cdot 3 = \int_1^5 f(x) dx$$

$$3 = \frac{1}{5-1} \int_1^5 f(x) dx$$

↑
Average Value of $f(x)$ over $[1, 5]$

Average Value of $f(x)$ on $[a, b]$.

$$= \frac{1}{b-a} \int_a^b f(x) dx$$

Ex. What's the Avg. Value of $f(x) = \cos(x)$ over $[0, 2\pi]$ |

$$= \frac{1}{2\pi - 0} \int_0^{2\pi} \cos(x) dx = \frac{1}{2\pi} (\sin(x)) \Big|_0^{2\pi} = \frac{1}{2\pi} (0 - 0) \\ = 0$$

Work:

Ex. Compute the work done while applying a force of $f(x) = x^3$ newtons over the range $[0, 3]$ feet.

$$W = \int_0^3 x^3 dx = \left. \frac{x^4}{4} \right|_0^3 = \frac{81}{4} = 20.25 \text{ N}\cdot\text{ft}$$