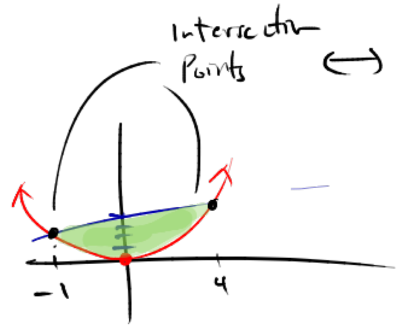
Thursday - Week 12

Area Calculation from HW.

Ex $f(x) = x^2 - 2x$

$f(x) = x + 4$ } Graph, Bound a region



$\longleftrightarrow$

$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

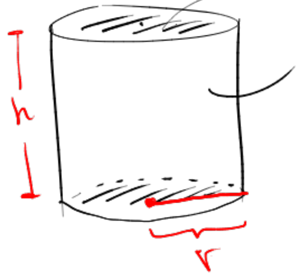
$$\int_{-1}^{4} x+4 - (x^2 - 2x)\,dx = \int_{-1}^{4} -x^2 + 3x + 4 \,dx = -\frac{x^3}{3} + \frac{3x^2}{2} + 4x \Big|_{-1}^{4}$$

$$= -\frac{64}{3} + \frac{48}{2} + 16 - \left[\frac{1}{3} + \frac{3}{2} - 4\right] = 20.8\overline{3}$$

More Optimization:

Vinyl LP Record @ $.25/in^2$

Record



Record Sleeves: / Cardboard: $.05/in^2$

Goal: ① Make a can with volume (that holds) $30\ in^3$. — constraint: $V=30$

② What dimensions will give the cheapest can?

$\Rightarrow$ Minimize cost.

Need a function for cost.

$$\text{Cost} = .25 \times (\text{Top Area}) + .05(\text{side area}) + .25 \times (\text{bottom area})$$

$$= .25(\pi r^2) + .05(h \times \text{circumf.}) + .25(\pi r^2)$$

$h \cdot 2\pi r$

$V=30$

$\pi r^2 h = 30$

$$h = \frac{30}{\pi r^2}$$

$$= .5\pi r^2 + .05\left(\frac{30}{\pi r^2} \cdot 2\pi r\right)$$

$60 \cdot .05 = 60 \cdot .01 \cdot 5$

$= .6 \times 5 = 3$

$$C(r) = .5\pi r^2 + \frac{3}{r}$$

cost is a function of r.

$\frac{3}{r} = 3r^{-1}$

$\downarrow$

$-3r^{-2}$

To find minimum of a function find $C'(r) = 0$

$$\boxed{C'(r) = \pi r - \frac{3}{r^2}} = 0$$

$\pi r = \frac{3}{r^2}$

$\pi r^3 = 3$

$r^3 = \frac{3}{\pi}$

$$r = \sqrt[3]{\frac{3}{\pi}} = .985$$

$$\Rightarrow h = \frac{30}{\pi(.985)^2} = 9.8$$

How do we know $r = .985$ gives the minimum cost?

What is the min. cost?

$3r^{-2}$

How do know: $C''(r) = \pi + 6r^{-3} = \pi + \frac{6}{r^3}$

$C''(.985) = \pi + \frac{6}{.9^3} > 0 \Rightarrow$ ∪ $\Rightarrow$ local min.

$$C(.985) = .5(\pi(.985)^2) + .05(2\pi(.985) \cdot 9.8) \approx \$4.55$$

top/bottom cost

Next App: Build Functions From their Derivatives

Idea:
$$f''(x) = 5x$$
$$f'(1) = 100$$
$$f(3) = 150$$

$$f'(x) = \int f''(x)\,dx = \int 5x\,dx = \frac{5x^2}{2} + C$$

$$f'(1) = 100 = \frac{5(1)^2}{2} + C \text{ so } C = 100 - \frac{5}{2} = 97.5$$

$$\boxed{f'(x) = \frac{5x^2}{2} + 97.5}$$

Repeat

$$f(x) = \int f'(x)\,dx = \int \frac{5x^2}{2} + 97.5\,dx$$

$$f(x) = \frac{5x^3}{6} + 97.5x + C$$

$$f(3) = 150 = \frac{5(3)^3}{6} + 97.5(3) + C$$

$$150 - 22.5 - 292.5 = C$$
$$150 - 315 = C$$
$$-165 = C$$

$(97.5)3$

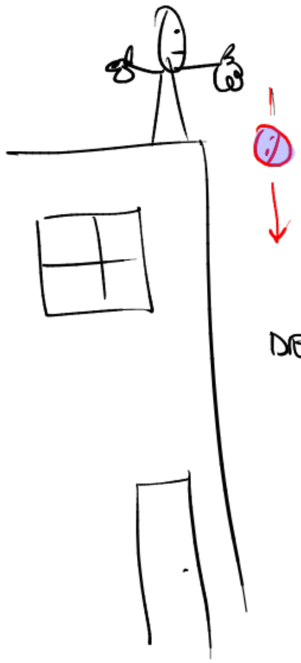
$(90 + 7 + .5)3$

$270 + 21 + 1.5$

292.5

$22.5 = \frac{45}{2}$ $\frac{5\cdot 3\cdot 3\cdot \cancel{3}}{2\cdot \cancel{6}}$

$$\boxed{f(x) = \frac{5x^3}{6} + 97.5x - 165}$$



Gravity $= f''(x) =$ "accelleration".

$f(x) =$ position/height

Drop ball from $100'$ high:

$$f''(x) = -32\ f/s^2$$

$f'(x) =$ velocity

$$f'(x) = \int -32\,dx = -32x + C$$

Drop $\Rightarrow f'(0) = 0$

$$f'(0) = 0 = -32(0) + C \Rightarrow C = 0$$

$$f'(x) = -32x$$

$$f(x) = \int -32x\,dx = -\frac{32x^2}{2} + C = -16x^2 + C$$

$$f(0) = 100 = -16(0)^2 + C \Rightarrow C = 100$$

$$f(x) = -16x^2 + 100$$

$f(1) = 84$

$f(2) = 36$

$f(3) = -44$

after it hits ground